✔ QUICK CHECK EXERCISES 8.2
(See page 522 for answers.)

1. (a) If $G'(x) = g(x)$, then
$$\int f(x)g(x)\,dx = f(x)G(x) - \_\_\_\_\_$$
(b) If $u = f(x)$ and $v = G(x)$, then the formula in part (a) can be written in the form $\int u\,dv = \_\_\_\_\_$.

2. Find an appropriate choice of u and dv for integration by parts of each integral. Do not evaluate the integral.
(a) $\int x \ln x\,dx$; $u = \_\_\_\_\_$, $dv = \_\_\_\_\_$
(b) $\int (x-2)\sin x\,dx$; $u = \_\_\_\_\_$, $dv = \_\_\_\_\_$
(c) $\int \sin^{-1} x\,dx$; $u = \_\_\_\_\_$, $dv = \_\_\_\_\_$
(d) $\int \dfrac{x}{\sqrt{x-1}}\,dx$; $u = \_\_\_\_\_$, $dv = \_\_\_\_\_$

3. Use integration by parts to evaluate the integral.
(a) $\int xe^{2x}\,dx$ (b) $\int \ln(x-1)\,dx$
(c) $\int_0^{\pi/6} x\sin 3x\,dx$

4. Use a reduction formula to evaluate $\int \sin^3 x\,dx$.

EXERCISE SET 8.2

1–40 Evaluate the integral.

1. $\int xe^{-2x}\,dx$
2. $\int xe^{3x}\,dx$
3. $\int x^2e^{x}\,dx$
4. $\int x^2e^{-2x}\,dx$
5. $\int x\sin 3x\,dx$
6. $\int x\cos 2x\,dx$
7. $\int x^2\cos x\,dx$
8. $\int x^2\sin x\,dx$
9. $\int x\ln x\,dx$
10. $\int \sqrt{x}\ln x\,dx$
11. $\int (\ln x)^2\,dx$
12. $\int \dfrac{\ln x}{\sqrt{x}}\,dx$
13. $\int \ln(3x-2)\,dx$
14. $\int \ln(x^2+4)\,dx$
15. $\int \sin^{-1} x\,dx$
16. $\int \cos^{-1}(2x)\,dx$
17. $\int \tan^{-1}(3x)\,dx$
18. $\int x\tan^{-1} x\,dx$
19. $\int e^x\sin x\,dx$
20. $\int e^{3x}\cos 2x\,dx$
21. $\int e^{ax}\sin bx\,dx$
22. $\int e^{-3\theta}\sin 5\theta\,d\theta$
23. $\int \sin(\ln x)\,dx$
24. $\int \cos(\ln x)\,dx$
25. $\int x\sec^2 x\,dx$
26. $\int x\tan^2 x\,dx$
27. $\int x^3e^{x^2}\,dx$
28. $\int \dfrac{xe^x}{(x+1)^2}\,dx$
29. $\int_0^2 xe^{2x}\,dx$
30. $\int_0^1 xe^{-5x}\,dx$
31. $\int_1^e x^2\ln x\,dx$
32. $\int_{\sqrt{e}}^{e} \dfrac{\ln x}{x^2}\,dx$
33. $\int_{-1}^{1} \ln(x+2)\,dx$
34. $\int_0^{\sqrt{3}/2} \sin^{-1} x\,dx$
35. $\int_2^4 \sec^{-1}\sqrt{\theta}\,d\theta$
36. $\int_1^2 x\sec^{-1} x\,dx$
37. $\int_0^{\pi} x\sin 2x\,dx$
38. $\int_0^{\pi} (x + x\cos x)\,dx$
39. $\int_1^3 \sqrt{x}\tan^{-1}\sqrt{x}\,dx$
40. $\int_0^2 \ln(x^2+1)\,dx$

41. In each part, evaluate the integral by making a u-substitution and then integrating by parts.
(a) $\int e^{\sqrt{x}}\,dx$ (b) $\int \cos\sqrt{x}\,dx$

42. Prove that tabular integration by parts gives the correct answer for
$$\int p(x)q(x)\,dx$$
where $p(x)$ is any *quadratic polynomial* and $q(x)$ is any function that can be repeatedly integrated.

43–46 Evaluate the integral using tabular integration by parts.

43. $\int (3x^2 - x + 2)e^{-x}\,dx$
44. $\int (x^2 + x + 1)\sin x\,dx$
45. $\int 4x^4\sin 2x\,dx$
46. $\int x^3\sqrt{2x+1}\,dx$

47. Evaluate the integral $\int \sin x\cos x\,dx$ using
(a) integration by parts (b) the substitution $u = \sin x$.

EXERCISE SET 8.3

1–52 Evaluate the integral.

1. $\int \cos^3 x \sin x\, dx$
2. $\int \sin^5 3x \cos 3x\, dx$
3. $\int \sin^2 5\theta\, d\theta$
4. $\int \cos^2 3x\, dx$
5. $\int \sin^3 a\theta\, d\theta$
6. $\int \cos^3 at\, dt$
7. $\int \sin ax \cos ax\, dx$
8. $\int \sin^3 x \cos^3 x\, dx$
9. $\int \sin^2 t \cos^3 t\, dt$
10. $\int \sin^3 x \cos^2 x\, dx$
11. $\int \sin^2 x \cos^2 x\, dx$
12. $\int \sin^2 x \cos^4 x\, dx$
13. $\int \sin 2x \cos 3x\, dx$
14. $\int \sin 3\theta \cos 2\theta\, d\theta$
15. $\int \sin x \cos(x/2)\, dx$
16. $\int \cos^{1/3} x \sin x\, dx$
17. $\int_0^{\pi/2} \cos^3 x\, dx$
18. $\int_0^{\pi/2} \sin^2 \frac{x}{2} \cos^2 \frac{x}{2}\, dx$
19. $\int_0^{\pi/3} \sin^4 3x \cos^3 3x\, dx$
20. $\int_{-\pi}^{\pi} \cos^2 5\theta\, d\theta$
21. $\int_0^{\pi/6} \sin 4x \cos 2x\, dx$
22. $\int_0^{2\pi} \sin^2 kx\, dx$
23. $\int \sec^2(2x-1)\, dx$
24. $\int \tan 5x\, dx$
25. $\int e^{-x} \tan(e^{-x})\, dx$
26. $\int \cot 3x\, dx$
27. $\int \sec 4x\, dx$
28. $\int \frac{\sec(\sqrt{x})}{\sqrt{x}}\, dx$
29. $\int \tan^2 x \sec^2 x\, dx$
30. $\int \tan^5 x \sec^4 x\, dx$
31. $\int \tan 4x \sec^4 4x\, dx$
32. $\int \tan^4 \theta \sec^4 \theta\, d\theta$
33. $\int \sec^5 x \tan^3 x\, dx$
34. $\int \tan^5 \theta \sec \theta\, d\theta$
35. $\int \tan^4 x \sec x\, dx$
36. $\int \tan^2 x \sec^3 x\, dx$
37. $\int \tan t \sec^3 t\, dt$
38. $\int \tan x \sec^5 x\, dx$
39. $\int \sec^4 x\, dx$
40. $\int \sec^5 x\, dx$
41. $\int \tan^3 4x\, dx$
42. $\int \tan^4 x\, dx$
43. $\int \sqrt{\tan x} \sec^4 x\, dx$
44. $\int \tan x \sec^{3/2} x\, dx$
45. $\int_0^{\pi/8} \tan^2 2x\, dx$
46. $\int_0^{\pi/6} \sec^3 2\theta \tan 2\theta\, d\theta$
47. $\int_0^{\pi/2} \tan^5 \frac{x}{2}\, dx$
48. $\int_0^{1/4} \sec \pi x \tan \pi x\, dx$
49. $\int \cot^3 x \csc^3 x\, dx$
50. $\int \cot^2 3t \sec 3t\, dt$
51. $\int \cot^3 x\, dx$
52. $\int \csc^4 x\, dx$

53. Let m, n be distinct nonnegative integers. Use Formulas (16)–(18) to prove:
(a) $\int_0^{2\pi} \sin mx \cos nx\, dx = 0$
(b) $\int_0^{2\pi} \cos mx \cos nx\, dx = 0$
(c) $\int_0^{2\pi} \sin mx \sin nx\, dx = 0.$

54. Evaluate the integrals in Exercise 53 when m and n denote the *same* nonnegative integer.

55. Find the arc length of the curve $y = \ln(\cos x)$ over the interval $[0, \pi/4]$.

56. Find the volume of the solid generated when the region enclosed by $y = \tan x$, $y = 1$, and $x = 0$ is revolved about the x-axis.

57. Find the volume of the solid that results when the region enclosed by $y = \cos x$, $y = \sin x$, $x = 0$, and $x = \pi/4$ is revolved about the x-axis.

58. The region bounded below by the x-axis and above by the portion of $y = \sin x$ from $x = 0$ to $x = \pi$ is revolved about the x-axis. Find the volume of the resulting solid.

59. Use Formula (27) to show that if the length of the equatorial line on a Mercator projection is L, then the vertical distance D between the latitude lines at $\alpha°$ and $\beta°$ on the same side of the equator (where $\alpha < \beta$) is

$$D = \frac{L}{2\pi} \ln \left| \frac{\sec \beta° + \tan \beta°}{\sec \alpha° + \tan \alpha°} \right|$$

60. Suppose that the equator has a length of 100 cm on a Mercator projection. In each part, use the result in Exercise 59 to answer the question.
(a) What is the vertical distance on the map between the equator and the line at 25° north latitude?
(b) What is the vertical distance on the map between New Orleans, Louisiana, at 30° north latitude and Winnipeg, Canada, at 50° north latitude?

FOCUS ON CONCEPTS

61. (a) Show that

$$\int \csc x\, dx = -\ln|\csc x + \cot x| + C$$

(b) Show that the result in part (a) can also be written as

$$\int \csc x\, dx = \ln|\csc x - \cot x| + C$$

EXERCISE SET 8.4 [C] CAS

1–26 Evaluate the integral.

1. $\int \sqrt{4-x^2}\,dx$
2. $\int \sqrt{1-4x^2}\,dx$
3. $\int \frac{x^2}{\sqrt{16-x^2}}\,dx$
4. $\int \frac{dx}{x^2\sqrt{9-x^2}}$
5. $\int \frac{dx}{(4+x^2)^2}$
6. $\int \frac{x^2}{\sqrt{5+x^2}}\,dx$
7. $\int \frac{\sqrt{x^2-9}}{x}\,dx$
8. $\int \frac{dx}{x^2\sqrt{x^2-16}}$
9. $\int \frac{3x^3}{\sqrt{1-x^2}}\,dx$
10. $\int x^3\sqrt{5-x^2}\,dx$
11. $\int \frac{dx}{x^2\sqrt{9x^2-4}}$
12. $\int \frac{\sqrt{1+t^2}}{t}\,dt$
13. $\int \frac{dx}{(1-x^2)^{3/2}}$
14. $\int \frac{dx}{x^2\sqrt{x^2+25}}$
15. $\int \frac{dx}{\sqrt{x^2-9}}$
16. $\int \frac{dx}{1+2x^2+x^4}$
17. $\int \frac{dx}{(4x^2-9)^{3/2}}$
18. $\int \frac{3x^3}{\sqrt{x^2-25}}\,dx$
19. $\int e^x\sqrt{1-e^{2x}}\,dx$
20. $\int \frac{\cos\theta}{\sqrt{2-\sin^2\theta}}\,d\theta$
21. $\int_0^1 5x^3\sqrt{1-x^2}\,dx$
22. $\int_0^{1/2} \frac{dx}{(1-x^2)^2}$

23. $\int_{\sqrt{2}}^{2} \frac{dx}{x^2\sqrt{x^2-1}}$
24. $\int_{\sqrt{2}}^{2} \frac{\sqrt{2x^2-4}}{x}\,dx$
25. $\int_1^3 \frac{dx}{x^4\sqrt{x^2+3}}$
26. $\int_0^3 \frac{x^3}{(3+x^2)^{5/2}}\,dx$

FOCUS ON CONCEPTS

27. The integral
$$\int \frac{x}{x^2+4}\,dx$$
can be evaluated either by a trigonometric substitution or by the substitution $u = x^2+4$. Do it both ways and show that the results are equivalent.

28. The integral
$$\int \frac{x^2}{x^2+4}\,dx$$
can be evaluated either by a trigonometric substitution or by algebraically rewriting the numerator of the integrand as $(x^2+4)-4$. Do it both ways and show that the results are equivalent.

29. Find the arc length of the curve $y = \ln x$ from $x = 1$ to $x = 2$.

30. Find the arc length of the curve $y = x^2$ from $x = 0$ to $x = 1$.

31. Find the area of the surface generated when the curve in Exercise 30 is revolved about the x-axis.

32. Find the volume of the solid generated when the region enclosed by $x = y(1-y^2)^{1/4}$, $y = 0$, $y = 1$, and $x = 0$ is revolved about the y-axis.

33–44 Evaluate the integral.

33. $\int \frac{dx}{x^2-4x+5}$
34. $\int \frac{dx}{\sqrt{2x-x^2}}$
35. $\int \frac{dx}{\sqrt{3+2x-x^2}}$
36. $\int \frac{dx}{16x^2+16x+5}$
37. $\int \frac{dx}{\sqrt{x^2-6x+10}}$
38. $\int \frac{x}{x^2+2x+2}\,dx$
39. $\int \sqrt{3-2x-x^2}\,dx$
40. $\int \frac{e^x}{\sqrt{1+e^x+e^{2x}}}\,dx$
41. $\int \frac{dx}{2x^2+4x+7}$
42. $\int \frac{2x+3}{4x^2+4x+5}\,dx$
43. $\int_1^2 \frac{dx}{\sqrt{4x-x^2}}$
44. $\int_0^4 \sqrt{x(4-x)}\,dx$

45–46 There is a good chance that your CAS will not be able to evaluate these integrals as stated. If this is so, make a substitution that converts the integral into one that your CAS can evaluate.

[C] 45. $\int \cos x \sin x\sqrt{1-\sin^4 x}\,dx$

[C] 46. $\int (x\cos x + \sin x)\sqrt{1+x^2\sin^2 x}\,dx$

FOCUS ON CONCEPTS

47. (a) Use the ***hyperbolic substitution*** $x = 3\sinh u$, the identity $\cosh^2 u - \sinh^2 u = 1$, and Theorem 7.9.4 to evaluate
$$\int \frac{dx}{\sqrt{x^2+9}}$$
(b) Evaluate the integral in part (a) using a trigonometric substitution and show that the result agrees with that obtained in part (a).

48. Use the hyperbolic substitution $x = \cosh u$, the identity $\sinh^2 u = \frac{1}{2}(\cosh 2u - 1)$, and the results referenced in Exercise 47 to evaluate
$$\int \sqrt{x^2-1}\,dx, \quad x \geq 1$$

EXERCISE SET 8.5 [C] CAS

1–8 Write out the form of the partial fraction decomposition. (Do not find the numerical values of the coefficients.)

1. $\dfrac{3x-1}{(x-3)(x+4)}$
2. $\dfrac{5}{x(x^2-4)}$
3. $\dfrac{2x-3}{x^3-x^2}$
4. $\dfrac{x^2}{(x+2)^3}$
5. $\dfrac{1-x^2}{x^3(x^2+2)}$
6. $\dfrac{3x}{(x-1)(x^2+6)}$
7. $\dfrac{4x^3-x}{(x^2+5)^2}$
8. $\dfrac{1-3x^4}{(x-2)(x^2+1)^2}$

9–32 Evaluate the integral.

9. $\displaystyle\int \frac{dx}{x^2-3x-4}$
10. $\displaystyle\int \frac{dx}{x^2-6x-7}$
11. $\displaystyle\int \frac{11x+17}{2x^2+7x-4}\,dx$
12. $\displaystyle\int \frac{5x-5}{3x^2-8x-3}\,dx$
13. $\displaystyle\int \frac{2x^2-9x-9}{x^3-9x}\,dx$
14. $\displaystyle\int \frac{dx}{x(x^2-1)}$

15. $\displaystyle\int \frac{x^2-8}{x+3}\,dx$
16. $\displaystyle\int \frac{x^2+1}{x-1}\,dx$
17. $\displaystyle\int \frac{3x^2-10}{x^2-4x+4}\,dx$
18. $\displaystyle\int \frac{x^2}{x^2-3x+2}\,dx$
19. $\displaystyle\int \frac{x^5+x^2+2}{x^3-x}\,dx$
20. $\displaystyle\int \frac{x^5-4x^3+1}{x^3-4x}\,dx$
21. $\displaystyle\int \frac{2x^2+3}{x(x-1)^2}\,dx$
22. $\displaystyle\int \frac{3x^2-x+1}{x^3-x^2}\,dx$
23. $\displaystyle\int \frac{2x^2-10x+4}{(x+1)(x-3)^2}\,dx$
24. $\displaystyle\int \frac{2x^2-2x-1}{x^3-x^2}\,dx$
25. $\displaystyle\int \frac{x^2}{(x+1)^3}\,dx$
26. $\displaystyle\int \frac{2x^2+3x+3}{(x+1)^3}\,dx$
27. $\displaystyle\int \frac{2x^2-1}{(4x-1)(x^2+1)}\,dx$
28. $\displaystyle\int \frac{dx}{x^3+2x}$
29. $\displaystyle\int \frac{x^3+3x^2+x+9}{(x^2+1)(x^2+3)}\,dx$
30. $\displaystyle\int \frac{x^3+x^2+x+2}{(x^2+1)(x^2+2)}\,dx$
31. $\displaystyle\int \frac{x^3-2x^2+2x-2}{x^2+1}\,dx$
32. $\displaystyle\int \frac{x^4+6x^3+10x^2+x}{x^2+6x+10}\,dx$

33–34 Evaluate the integral by making a substitution that converts the integrand to a rational function.

33. $\displaystyle\int \frac{\cos\theta}{\sin^2\theta+4\sin\theta-5}\,d\theta$
34. $\displaystyle\int \frac{e^t}{e^{2t}-4}\,dt$

35. Find the volume of the solid generated when the region enclosed by $y=x^2/(9-x^2)$, $y=0$, $x=0$, and $x=2$ is revolved about the x-axis.

36. Find the area of the region under the curve $y=1/(1+e^x)$, over the interval $[-\ln 5, \ln 5]$. [*Hint:* Make a substitution that converts the integrand to a rational function.]

37–38 Use a CAS to evaluate the integral in two ways: (i) integrate directly; (ii) use the CAS to find the partial fraction decomposition and integrate the decomposition. Integrate by hand to check the results.

[C] 37. $\displaystyle\int \frac{x^2+1}{(x^2+2x+3)^2}\,dx$

[C] 38. $\displaystyle\int \frac{x^5+x^4+4x^3+4x^2+4x+4}{(x^2+2)^3}\,dx$

39–40 Integrate by hand and check your answers using a CAS.

[C] 39. $\displaystyle\int \frac{dx}{x^4-3x^3-7x^2+27x-18}$

[C] 40. $\displaystyle\int \frac{dx}{16x^3-4x^2+4x-1}$

FOCUS ON CONCEPTS

41. Show that

$$\int_0^1 \frac{x}{x^4+1}\,dx = \frac{\pi}{8}$$

42. Use partial fractions to derive the integration formula

$$\int \frac{1}{a^2-x^2}\,dx = \frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right| + C$$

43. Suppose that ax^2+bx+c is a quadratic polynomial and that the integration

$$\int \frac{1}{ax^2+bx+c}\,dx$$

produces a function with no inverse tangent terms. What does this tell you about the roots of the polynomial?

44. Suppose that ax^2+bx+c is a quadratic polynomial and that the integration

$$\int \frac{1}{ax^2+bx+c}\,dx$$

produces a function with neither logarithmic nor inverse tangent terms. What does this tell you about the roots of the polynomial?

45. Does there exist a quadratic polynomial ax^2+bx+c such that the integration

$$\int \frac{x}{ax^2+bx+c}\,dx$$

produces a function with no logarithmic terms? If so, give an example; if not, explain why no such polynomial can exist.

✔ QUICK CHECK EXERCISES 8.8

(See page 580 for answers.)

1. In each part, determine whether the integral is improper, and if so, explain why. Do not evaluate the integrals.
   (a) $\int_{\pi/4}^{3\pi/4} \cot x\,dx$ (b) $\int_{\pi/4}^{\pi} \cot x\,dx$
   (c) $\int_0^{+\infty} \frac{1}{x^2+1}\,dx$ (d) $\int_1^{+\infty} \frac{1}{x^2-1}\,dx$
   (e) $\int_0^{\pi/4} \tan x\,dx$
2. Express each improper integral in Quick Check Exercise 1 in terms of one or more appropriate limits. Do not evaluate the limits.
3. The improper integral
$$\int_1^{+\infty} x^{-p}\,dx$$
converges to ________ provided ________.
4. Evaluate the integrals that converge.
   (a) $\int_0^{+\infty} e^{-x}\,dx$ (b) $\int_0^{+\infty} e^{x}\,dx$
   (c) $\int_0^1 \frac{1}{x^3}\,dx$ (d) $\int_0^1 \frac{1}{\sqrt[3]{x^2}}\,dx$

EXERCISE SET 8.8

Graphing Utility [C] CAS

1. In each part, determine whether the integral is improper, and if so, explain why.
   (a) $\int_1^5 \frac{dx}{x-3}$ (b) $\int_1^5 \frac{dx}{x+3}$ (c) $\int_0^1 \ln x\,dx$
   (d) $\int_1^{+\infty} e^{-x}\,dx$ (e) $\int_{-\infty}^{+\infty} \frac{dx}{\sqrt[3]{x-1}}$ (f) $\int_0^{\pi/4} \tan x\,dx$
2. In each part, determine all values of p for which the integral is improper.
   (a) $\int_0^1 \frac{dx}{x^p}$ (b) $\int_1^2 \frac{dx}{x-p}$ (c) $\int_0^1 e^{-px}\,dx$

3–30 Evaluate the integrals that converge.

3. $\int_0^{+\infty} e^{-2x}\,dx$
4. $\int_{-1}^{+\infty} \frac{x}{1+x^2}\,dx$
5. $\int_3^{+\infty} \frac{2}{x^2-1}\,dx$
6. $\int_0^{+\infty} xe^{-x^2}\,dx$
7. $\int_e^{+\infty} \frac{1}{x\ln^3 x}\,dx$
8. $\int_2^{+\infty} \frac{1}{x\sqrt{\ln x}}\,dx$
9. $\int_{-\infty}^{0} \frac{dx}{(2x-1)^3}$
10. $\int_{-\infty}^{3} \frac{dx}{x^2+9}$
11. $\int_{-\infty}^{0} e^{3x}\,dx$
12. $\int_{-\infty}^{0} \frac{e^x\,dx}{3-2e^x}$
13. $\int_{-\infty}^{+\infty} x\,dx$
14. $\int_{-\infty}^{+\infty} \frac{x}{\sqrt{x^2+2}}\,dx$
15. $\int_{-\infty}^{+\infty} \frac{x}{(x^2+3)^2}\,dx$
16. $\int_{-\infty}^{+\infty} \frac{e^{-t}}{1+e^{-2t}}\,dt$
17. $\int_0^4 \frac{dx}{(x-4)^2}$
18. $\int_0^8 \frac{dx}{\sqrt[3]{x}}$
19. $\int_0^{\pi/2} \tan x\,dx$
20. $\int_0^4 \frac{dx}{\sqrt{4-x}}$
21. $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$
22. $\int_{-3}^{1} \frac{x\,dx}{\sqrt{9-x^2}}$
23. $\int_{\pi/3}^{\pi/2} \frac{\sin x}{\sqrt{1-2\cos x}}\,dx$
24. $\int_0^{\pi/4} \frac{\sec^2 x}{1-\tan x}\,dx$
25. $\int_0^3 \frac{dx}{x-2}$
26. $\int_{-2}^{2} \frac{dx}{x^2}$
27. $\int_{-1}^{8} x^{-1/3}\,dx$
28. $\int_0^1 \frac{dx}{(x-1)^{2/3}}$
29. $\int_0^{+\infty} \frac{1}{x^2}\,dx$
30. $\int_1^{+\infty} \frac{dx}{x\sqrt{x^2-1}}$

31–34 Make the u-substitution and evaluate the resulting definite integral.

31. $\int_0^{+\infty} \frac{e^{-\sqrt{x}}}{\sqrt{x}}\,dx;\ u=\sqrt{x}$ [*Note:* $u \to +\infty$ as $x \to +\infty$.]
32. $\int_{12}^{+\infty} \frac{dx}{\sqrt{x}(x+4)};\ u=\sqrt{x}$
33. $\int_0^{+\infty} \frac{e^{-x}}{\sqrt{1-e^{-x}}}\,dx;\ u=1-e^{-x}$
[*Note:* $u \to 1$ as $x \to +\infty$.]
34. $\int_0^{+\infty} \frac{e^{-x}}{\sqrt{1-e^{-2x}}}\,dx;\ u=e^{-x}$

35–36 Express the improper integral as a limit, and then evaluate that limit with a CAS. Confirm the answer by evaluating the integral directly with the CAS.

[C] 35. $\int_0^{+\infty} e^{-x}\cos x\,dx$
[C] 36. $\int_0^{+\infty} xe^{-3x}\,dx$

Exercise Set 8.2 (Page 520)

1. $-e^{-2x}\left(\frac{x}{2}+\frac{1}{4}\right)+C$ 3. $x^2e^x-2xe^x+2e^x+C$
5. $-\frac{1}{3}x\cos 3x+\frac{1}{9}\sin 3x+C$ 7. $x^2\sin x+2x\cos x-2\sin x+C$
9. $\frac{x^2}{2}\ln x-\frac{x^2}{4}+C$ 11. $x(\ln x)^2-2x\ln x+2x+C$
13. $x\ln(3x-2)-x-\frac{2}{3}\ln(3x-2)+C$ 15. $x\sin^{-1}x+\sqrt{1-x^2}+C$
17. $x\tan^{-1}(3x)-\frac{1}{6}\ln(1+9x^2)+C$ 19. $\frac{1}{2}e^x(\sin x-\cos x)+C$
21. $\frac{e^{ax}}{a^2+b^2}(a\sin bx-b\cos bx)+C$
23. $(x/2)[\sin(\ln x)-\cos(\ln x)]+C$ 25. $x\tan x+\ln|\cos x|+C$
27. $\frac{1}{2}x^2e^{x^2}-\frac{1}{2}e^{x^2}+C$ 29. $\frac{1}{4}(3e^4+1)$ 31. $(2e^3+1)/9$
33. $3\ln 3-2$ 35. $\frac{5\pi}{6}-\sqrt{3}+1$ 37. $-\pi/2$
39. $\frac{1}{3}\left(2\sqrt{3}\pi-\frac{\pi}{2}-2+\ln 2\right)$
41. (a) $2(\sqrt{x}-1)e^{\sqrt{x}}+C$ (b) $2\sqrt{x}\sin\sqrt{x}+2\cos\sqrt{x}+C$
43. $-(3x^2+5x+7)e^{-x}+C$
45. $(4x^3-6x)\sin 2x-(2x^4-6x^2+3)\cos 2x+C$
47. (a) $\frac{1}{2}\sin^2 x+C$ (b) $\frac{1}{2}\sin^2 x+C$
49. (a) $A=1$ (b) $V=\pi(e-2)$ 51. $V=2\pi^2$ 53. $\pi^3-6\pi$
55. (a) $-\frac{1}{4}\sin^3 x\cos x-\frac{3}{8}\sin x\cos x+\frac{3}{8}x+C$ (b) 8/15
59. (a) $\frac{1}{3}\tan^3 x-\tan x+x+C$ (b) $\frac{1}{3}\sec^2 x\tan x+\frac{2}{3}\tan x+C$
(c) $x^3e^x-3x^2e^x+6xe^x-6e^x+C$
63. $(x+1)\ln(x+1)-x+C$ 65. $\frac{1}{2}(x^2+1)\tan^{-1}x-\frac{1}{2}x+C$

Exercise Set 8.3 (Page 529)

1. $-\frac{1}{4}\cos^4 x+C$ 3. $\frac{\theta}{2}-\frac{1}{20}\sin 10\theta+C$
5. $\frac{1}{3a}\cos^3 a\theta-\cos a\theta+C$ 7. $\frac{1}{2a}\sin^2 ax+C$
9. $\frac{1}{3}\sin^3 t-\frac{1}{5}\sin^5 t+C$ 11. $\frac{1}{8}x-\frac{1}{32}\sin 4x+C$
13. $-\frac{1}{10}\cos 5x+\frac{1}{2}\cos x+C$ 15. $-\frac{1}{3}\cos(3x/2)-\cos(x/2)+C$
17. 2/3 19. 0 21. 7/24 23. $\frac{1}{2}\tan(2x-1)+C$
25. $\ln|\cos(e^{-x})|+C$ 27. $\frac{1}{4}\ln|\sec 4x+\tan 4x|+C$
29. $\frac{1}{3}\tan^3 x+C$ 31. $\frac{1}{16}\sec^4 4x+C$ 33. $\frac{1}{7}\sec^7 x-\frac{1}{5}\sec^5 x+C$
35. $\frac{1}{4}\sec^3 x\tan x-\frac{5}{8}\sec x\tan x+\frac{3}{8}\ln|\sec x+\tan x|+C$
37. $\frac{1}{3}\sec^3 t+C$ 39. $\tan x+\frac{1}{3}\tan^3 x+C$
41. $\frac{1}{8}\tan^2 4x+\frac{1}{4}\ln|\cos 4x|+C$ 43. $\frac{2}{3}\tan^{3/2}x+\frac{2}{7}\tan^{7/2}x+C$
45. $\frac{1}{2}-\frac{\pi}{8}$ 47. $-\frac{1}{2}+\ln 2$ 49. $-\frac{1}{5}\csc^5 x+\frac{1}{3}\csc^3 x+C$
51. $-\frac{1}{2}\csc^2 x-\ln|\sin x|+C$ 55. $L=\ln(\sqrt{2}+1)$ 57. $V=\pi/2$
63. $-\frac{1}{\sqrt{a^2+b^2}}\ln\left[\frac{\sqrt{a^2+b^2}+a\cos x-b\sin x}{a\sin x+b\cos x}\right]+C$
65. (a) $\frac{2}{3}$ (b) $3\pi/16$ (c) $\frac{8}{15}$ (d) $5\pi/32$

Exercise Set 8.4 (Page 535)

1. $2\sin^{-1}(x/2)+\frac{1}{2}x\sqrt{4-x^2}+C$ 3. $8\sin^{-1}\left(\frac{x}{4}\right)-\frac{x\sqrt{16-x^2}}{2}+C$
5. $\frac{1}{16}\tan^{-1}(x/2)+\frac{x}{8(4+x^2)}+C$ 7. $\sqrt{x^2-9}-3\sec^{-1}(x/3)+C$
9. $-(x^2+2)\sqrt{1-x^2}+C$ 11. $\frac{\sqrt{9x^2-4}}{4x}+C$ 13. $\frac{x}{\sqrt{1-x^2}}+C$
15. $\ln|\sqrt{x^2-9}+x|+C$ 17. $\frac{-x}{9\sqrt{4x^2-9}}+C$
19. $\frac{1}{2}\sin^{-1}(e^x)+\frac{1}{2}e^x\sqrt{1-e^{2x}}+C$ 21. 2/3 23. $(\sqrt{3}-\sqrt{2})/2$
25. $\frac{10\sqrt{3}+18}{243}$ 27. $\frac{1}{2}\ln(x^2+4)+C$
29. $L=\sqrt{5}-\sqrt{2}+\ln\frac{2+2\sqrt{2}}{1+\sqrt{5}}$ 31. $S=\frac{\pi}{32}[18\sqrt{5}-\ln(2+\sqrt{5})]$
33. $\tan^{-1}(x-2)+C$ 35. $\sin^{-1}\left(\frac{x-1}{2}\right)+C$
37. $\ln(x-3+\sqrt{(x-3)^2+1})+C$
39. $2\sin^{-1}\left(\frac{x+1}{2}\right)+\frac{1}{2}(x+1)\sqrt{3-2x-x^2}+C$
41. $\frac{1}{\sqrt{10}}\tan^{-1}\sqrt{\frac{2}{5}}(x+1)+C$ 43. $\pi/6$
45. $u=\sin^2 x$, $\frac{1}{2}\int\sqrt{1-u^2}du$
$=\frac{1}{4}[\sin^2 x\sqrt{1-\sin^4 x}+\sin^{-1}(\sin^2 x)]+C$
47. (a) $\sinh^{-1}(x/3)+C$ (b) $\ln\left(\frac{\sqrt{x^2+9}}{3}+\frac{x}{3}\right)+C$

Exercise Set 8.5 (Page 543)

1. $\frac{A}{x-3}+\frac{B}{x+4}$ 3. $\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x-1}$
5. $\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x^3}+\frac{Dx+E}{x^2+2}$ 7. $\frac{Ax+B}{x^2+5}+\frac{Cx+D}{(x^2+5)^2}$
9. $\frac{1}{5}\ln\left|\frac{x-4}{x+1}\right|+C$ 11. $\frac{5}{2}\ln|2x-1|+3\ln|x+4|+C$
13. $\ln\left|\frac{x(x+3)^2}{x-3}\right|+C$ 15. $\frac{x^2}{2}-3x+\ln|x+3|+C$
17. $3x+12\ln|x-2|-\frac{2}{x-2}+C$
19. $x+\frac{x^3}{3}+\ln\left|\frac{(x-1)^2(x+1)}{x^2}\right|+C$
21. $3\ln|x|-\ln|x-1|-\frac{5}{x-1}+C$
23. $\frac{2}{x-3}+\ln|x-3|+\ln|x+1|+C$
25. $\frac{2}{x+1}-\frac{1}{2(x+1)^2}+\ln|x+1|+C$
27. $-\frac{7}{34}\ln|4x-1|+\frac{6}{17}\ln(x^2+1)+\frac{3}{17}\tan^{-1}x+C$
29. $3\tan^{-1}x+\frac{1}{2}\ln(x^2+3)+C$
31. $\frac{x^2}{2}-2x+\frac{1}{2}\ln(x^2+1)+C$ 33. $\frac{1}{6}\ln\left(\frac{1-\sin\theta}{5+\sin\theta}\right)+C$
35. $V=\pi\left(\frac{19}{5}-\frac{9}{4}\ln 5\right)$ 37. $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right)+\frac{1}{x^2+2x+3}+C$
39. $\frac{1}{8}\ln|x-1|-\frac{1}{5}\ln|x-2|+\frac{1}{12}\ln|x-3|-\frac{1}{120}\ln|x+3|+C$

Exercise Set 8.8 (Page 576)

1. (a) improper; infinite discontinuity at $x=3$ (b) not improper
(c) improper; infinite discontinuity at $x=0$
(d) improper; infinite interval of integration
(e) improper; infinite interval of integration and infinite discontinuity at $x=1$ (f) not improper
3. 1/2 5. $\ln 2$ 7. $\frac{1}{2}$ 9. $-\frac{1}{4}$ 11. $\frac{1}{3}$ 13. divergent 15. 0
17. divergent 19. divergent 21. $\pi/2$ 23. 1 25. divergent
27. $\frac{9}{2}$ 29. divergent 31. 2 33. 2 35. $\frac{1}{2}$
37. (a) 2.726585 (b) 2.804364 (c) 0.219384 (d) 0.504067 39. 12
41. -1 43. $\frac{1}{3}$ 45. (a) $V=\pi/2$ (b) $S=\pi[\sqrt{2}+\ln(1+\sqrt{2})]$
47. (b) $1/e$ (c) It is convergent. 53. $\frac{2\pi NI}{kr}\left(1-\frac{a}{\sqrt{r^2+a^2}}\right)$
55. (b) 2.4×10^7 mi.lb 57. (a) $\frac{1}{s^2}$ (b) $\frac{2}{s^3}$ (c) $\frac{e^{-3s}}{s}$ 61. (a) 1.047
65. 1.809 67. (a) $\Gamma(1)=1$ (c) $\Gamma(2)=1$, $\Gamma(3)=2$, $\Gamma(4)=6$
69. (b) 1.37078 seconds