April 21, 2022

TECHNIQUES OF INTEGRATION

REVIEW

$\cos^2 x = \frac{1}{2}(1+\cos 2x$

$\sin^2 x = \frac{1}{2}(1-\cos 2x)$

$\int u\,dv = uv - \int v\,du$

Integ. by parts

$\int x^2 \cos x$ $\qquad \int e^x (x^2+2x+1)\,dx$

$u = x^2 \qquad dv = \cos x$

$du = 2x\,dx \qquad v = \sin x$

$= x^2 \sin x - \int 2x \sin x\,dx$

$x^2 \sin x \pm \left[\pm 2x \cos x \mp \int \pm 2\cos x\,dx \right.$

$u = 2x \qquad dv = \sin x\,dx$

$du = 2 \qquad v = -\cos x\,dx$

$x^2 \sin x + 2x \cos x - 2\sin x + C$

$$\int \sin 4x \cos 8x \, dx$$

sum + product identity

$$\frac{1}{2}\int \left[\sin 12x + \sin(-4x)\right] dx$$

$$\frac{1}{2}\int \left(\sin 12x - \sin 4x\right) dx$$

$$\frac{1}{2}\left[-\frac{1}{12}\cos 12x + \frac{1}{4}\cos 4x\right] + C$$

$$\int \sin^3 x \cos^2 x \, dx$$

$$\int \cos^2 x \cdot \sin^2 x \cdot \sin x \, dx$$

$$\int \cos^2 x \left(1 - \cos^2 x\right) \sin x \, dx$$

$$\int \left(\cos^2 x - \cos^4 dx\right) \sin x \, dx$$

$u = \cos x$

$du = -\sin x \, dx$

$$\int \left(u^2 - u^4\right) \cancel{\sin x} \cdot \frac{du}{-\cancel{\sin x}}$$

$$\int \sin^2 x \cos^4 x \, dx$$

$$\int \sin^2 x \cdot \left(\cos^2 x\right)^2 dx$$

$$\int \frac{1}{2}\left(1 - \cos 2x\right) \cdot \left[\frac{1}{2}\left(1 + \cos 2x\right)\right]^2$$

$$\frac{1}{8}\int \left(1 - \cos 2x\right)\left(1 + \cos 2x\right)\left(1 + \cos 2x\right)$$

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Trig Substitution

$$\int \frac{4x^2}{\sqrt{1-9x^2}}\,dx \qquad \int \frac{2x^2}{4x^2+1}\,dx$$

$u = \sin\theta$

$$\int \frac{\cos\theta}{\sin^2\theta}\,d\theta \qquad \int \frac{\sec\theta}{\tan^2\theta}\,d\theta \quad \frac{\frac{1}{\cos\theta}}{\frac{\sin^2\theta}{\cos^2\theta}}$$