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TRIGONOMETRIC INTEGRALS

Use sum & prod. identity

$$\int \sin 4x \cos 3x \, dx$$

$$\frac{1}{2}\int \left(\sin \overset{4x+3x}{7x} + \sin \overset{4x-3x}{x}\right) dx$$

$$\frac{1}{2}\left[-\frac{1}{7}\cos 7x - \cos x\right] + C$$

$$\boxed{-\frac{1}{14}\cos 7x - \frac{1}{2}\cos x + C}$$

$$\int \sin^5 x \, dx$$

$$\int \sin x \cdot \sin^4 x \, dx$$

$$\int \sin x \cdot \left(\sin^2 x\right)^2 dx$$

$$\int \sin x \left(1 - \cos^2 x\right)^2 dx$$

$$u = \cos x$$
$$du = -\sin x \, dx$$

$$\int \cancel{\sin x} \underbrace{\left(1 - u^2\right)^2}_{(1-u^2)(1-u^2)} \frac{du}{-\cancel{\sin x}}$$

$$+\int -\left(1 + 2u^2 - u^4\right) du$$

$$= -u + \frac{2}{3}u^3 - \frac{1}{5}u^5 + C$$

$$= -\cos x + \frac{2}{3}\cos^3 x - \frac{1}{5}\cos^5 x + C$$

If one term has an <u>odd</u> power:

1) Split $\sin x$ or $\cos x$ off the odd powered term
2) Rewrite the remaining term as $(\sin^2 x)^p$ or $(\cos^2 x)^p$
3) Use $\sin^2 x + \cos^2 x = 1$ to rewrite squared term
4) U-sub, cancel, & integrate

$$(x + 3y)^5$$

$$x^5(3y)^0 \quad x^4(3y)^1 \quad x^3(3y)^2 \quad x^2$$

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$$\int \cos^4 x \, dx$$

$$\int (\cos^2 x)^2 \, dx$$

$$\int \left[\frac{1}{2}(1+\cos 2x)\right]^2 dx$$

$$(1+\cos 2x)(1+\cos 2x)$$

$$\frac{1}{4}\int (1 + 2\cos 2x + \cos^2 2x)\, dx$$

$$\frac{1}{4}\left[\int 1\, dx + \int 2\cos 2x\, dx + \int \cos^2 2x\, dx\right.$$

$$\frac{1}{4}\left[x + \sin 2x + \frac{1}{2}\int (1 + \cos 4x)\, dx\right.$$

$$\frac{1}{4}\left[x + \sin 2x + \frac{1}{2}\left[x + \frac{1}{4}\sin 4x\right]\right] + C$$

$$\frac{1}{4}\left[x + \sin 2x + \frac{1}{2}x + \frac{1}{8}\sin 4x\right] + C$$

$$\frac{1}{4}\left[\frac{3}{2}x + \sin 2x + \frac{1}{8}\sin 4x\right] + C$$

$$\boxed{\frac{3}{8}x + \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x + C}$$

$$\cos 2x = 1 - 2\sin^2 x \qquad \cos 2x = 2\cos^2 x - 1$$

$$2\sin^2 x = 1 - \cos 2x \qquad 1 + \cos 2x = 2\cos^2 x$$

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x) \qquad \frac{1}{2}(1 + \cos \underset{4x}{2x}) = \cos^2 \underset{2x}{x}$$

Both powers even

1) Rewrite as powers of $(\sin^2 x)^P$ or $(\cos^2 x)^P$
2) Use a double angle identity with $\sin^2 x$ or $\cos^2 x$
3) FOIL + integrate each term separately.

12/ $\int \sin^2 x \cos^4 x \, dx$

$\int \sin^2 x \left(\cos^2 x\right)^2 dx$

$\int \frac{1}{2}\left(1 - \cos 2x\right) \cdot \left[\frac{1}{2}\left(1 + \cos 2x\right)\right]^2$

$\frac{1}{8} \int \left(1 - \cos 2x\right)\left(1 + \cos 2x\right)\left(1 + \cos 2x\right) dx$

$\frac{1}{8} \int \left(1 - \cos^2 2x\right)\left(1 + \cos 2x\right) dx$

$\frac{1}{8} \int \left(\sin^2 2x\right)\left(1 + \cos 2x\right) dx$

$\frac{1}{8} \int \sin^2 \underline{2x} \, dx + \frac{1}{8} \int \sin^2 2x \cos 2x \, dx$

$u = \sin 2x$

↑ Double Angle identity

$$\int \sin^6 2x \cos^3 2x \, dx$$

$$\int \sin^6 2x \cdot \cos^2 2x \cdot \cos 2x \, dx$$

$$\int \sin^6 2x \cdot (1 - \sin^2 2x) \cos 2x$$

$u = \sin 2x$

$du = \cos 2x \cdot 2$

$$\int u^6 (1 - u^2) \cancel{\cos 2x} \cdot \frac{du}{2 \cancel{\cos 2x}}$$

$$\frac{1}{2} \int (u^6 - u^8) \, du$$

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