TRIG SUBSTITUTION

$a^2 - x^2$	$x = a\sin\theta$
$x^2 + a^2$	$x = a\tan\theta$
$x^2 - a^2$	$x = a\sec\theta$

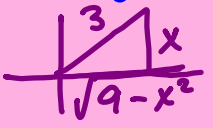
$16 - x^2$ $\quad x = 4\sin\theta$

($16 = a^2$)

$$\int \frac{x^2}{\sqrt{9-x^2}}\,dx \qquad x = 3\sin\theta \qquad dx = 3\cos\theta\,d\theta$$

$\frac{x}{3} = \sin\theta$

$\theta = \sin^{-1}\left(\frac{x}{3}\right)$



$$\int \frac{9\sin^2\theta}{\sqrt{9-9\sin^2\theta}} \cdot 3\cos\theta\,d\theta$$

$$\frac{27}{3}\int \frac{\sin^2\theta\cdot\cos\theta}{\sqrt{1-\sin^2\theta}}\,d\theta$$

$$9\int \frac{\sin^2\theta\cos\theta}{\sqrt{\cos^2\theta}}\,d\theta$$

$$9\int \frac{\sin^2\theta\cos\theta}{\cos\theta}\,d\theta$$

$$9\int \frac{1}{2}(1-\cos 2\theta)\,d\theta$$

$$\frac{9}{2}\left[\theta - \frac{1}{2}\sin 2\theta\right] + C$$

$$\frac{9}{2}\theta - \frac{9}{4}\sin 2\theta + C$$

$$\frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right) - \frac{9}{2}\cdot\sin\theta\cos\theta + C$$

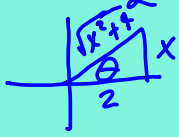
$$\frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right) - \frac{9}{2}\left(\frac{x}{3}\right)\left(\frac{\sqrt{9-x^2}}{3}\right) + C$$

$$\boxed{\frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right) - \frac{1}{2}x\sqrt{9-x^2} + C}$$

$$\int_a^b \frac{1}{x^2\sqrt{x^2+4}}\,dx$$ ← Change this!

$x = 2\tan\theta \longrightarrow \tan\theta = \frac{x}{2}$

$dx = 2\sec^2\theta\,d\theta$



$\sin\theta = \frac{x}{\sqrt{x^2+4}}$

$$\int_c^d \frac{1}{4\tan^2\theta\sqrt{4\tan^2\theta+4}}\,2\sec^2\theta\,d\theta$$

$$\frac{2}{4\cdot 2}\int \frac{\sec^2\theta}{\tan^2\theta\sqrt{\tan^2\theta+1}}\,d\theta$$

$$\frac{1}{4}\int \frac{\sec^2\theta}{\tan^2\theta\sqrt{\sec^2\theta}}\,d\theta$$

$\sec\theta$

$$\frac{1}{4}\int \frac{\sec\theta}{\tan^2\theta}\,d\theta$$

$$\frac{1}{4}\int \frac{\frac{1}{\cos\theta}}{\frac{\sin^2\theta}{\cos^2\theta}}\,d\theta$$

$$\frac{1}{4}\int \frac{1}{\cos\theta}\cdot\frac{\cos^2\theta}{\sin^2\theta}\,d\theta$$

$$\frac{1}{4}\int \frac{\cos\theta}{\sin^2\theta}\,d\theta$$

$u = \sin\theta$

$du = \cos\theta\,d\theta$

$$\frac{1}{4}\int_e^f \frac{\cos\theta}{u^2}\cdot\frac{du}{\cos\theta}$$

$$\frac{1}{4}\int \frac{1}{u^2}\,du$$

$$\frac{1}{4}\int u^{-2}\,du$$

$$\frac{1}{4}\,\frac{u^{-1}}{-1} + C$$

$$\frac{-1}{4u} + C$$

$$\frac{-1}{4\sin\theta} + C$$

$$\frac{-1}{4\cdot\frac{x}{\sqrt{x^2+4}}} + C$$

$$\boxed{-\frac{\sqrt{x^2+4}}{4x} + C}$$

$$\int \frac{1}{\sec^2\theta}\, d\theta = \int \cos^2\theta\, d\theta = \int \frac{1}{2}(1+\cos 2\theta)\, d\theta$$

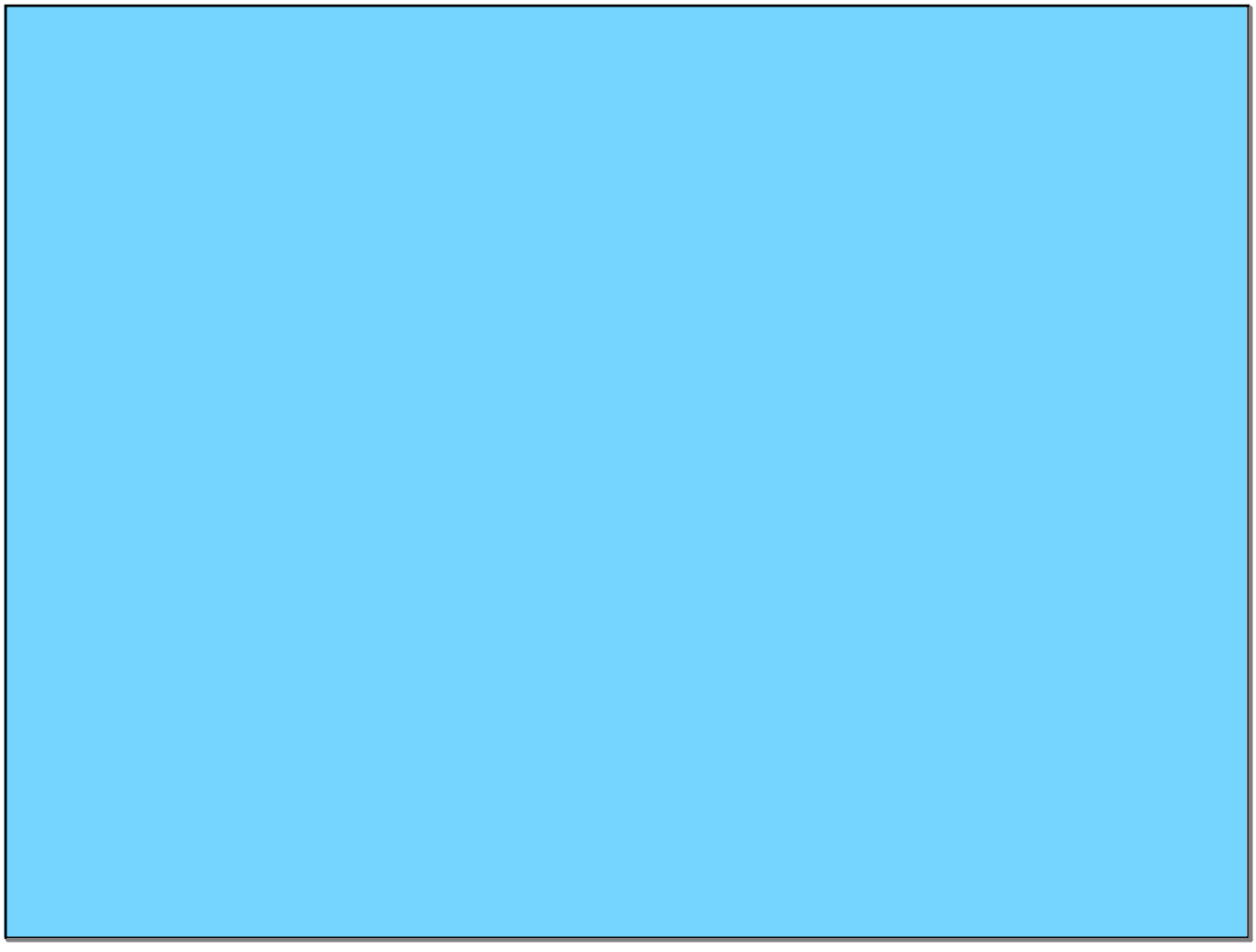
$$\int \tan^2\theta\, d\theta = \int (\sec^2\theta - 1)\, d\theta \longrightarrow 1+\tan^2\theta = \sec^2\theta$$

$$= \tan\theta - \theta + C$$

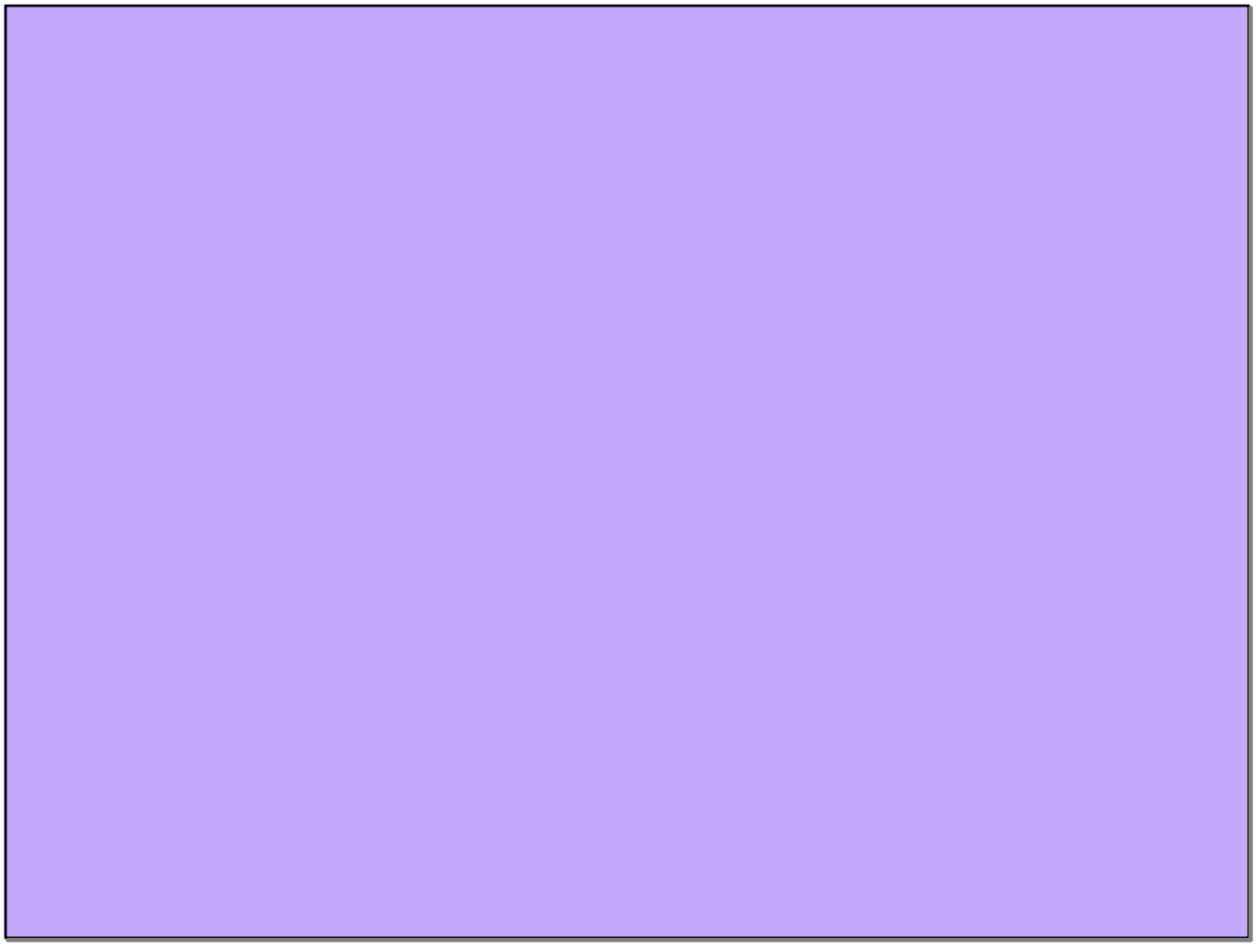
↑ Draw a pic

↑ Inv trig func.

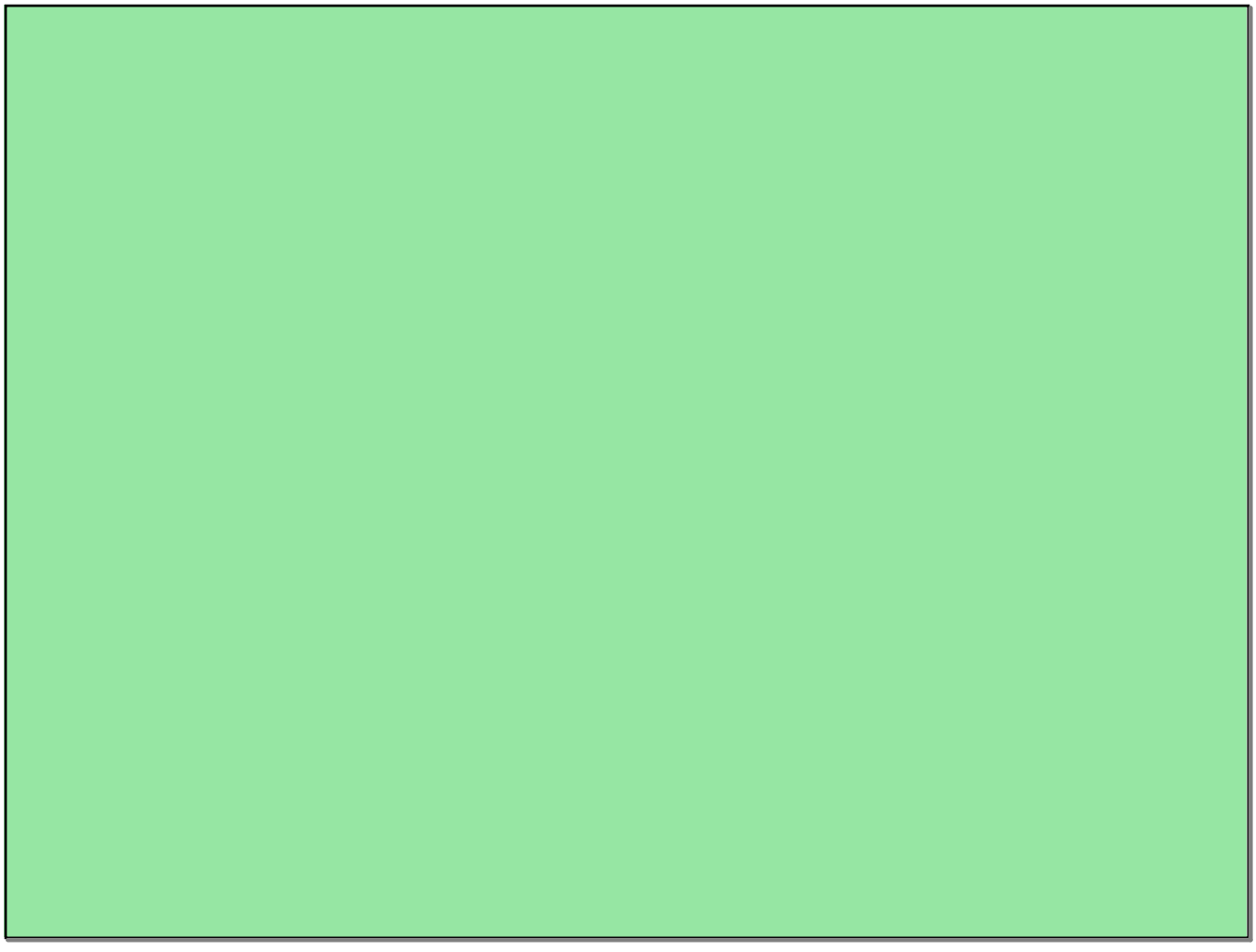
April 11, 2022



April 11, 2022



April 11, 2022



April 11, 2022