May 4, 2022

Semester 2 Review Day 2

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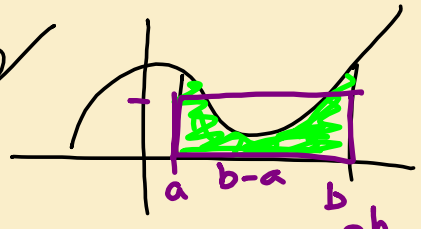


$$f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

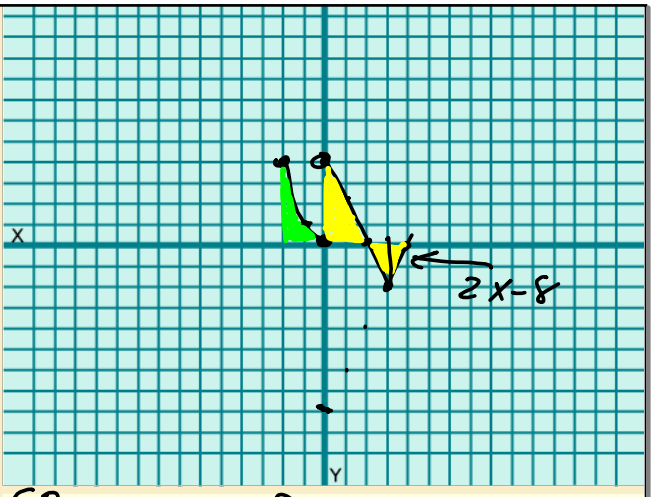
Find area

$$f(x)=\begin{cases} x^2 & -2\le x\le 0 \\ 2|x-3|-2 & 0<x\le 4 \end{cases}$$

$a=-2$ to $b=4$

$2(x-3)-2 = 2x-6-2$
$= 2x-8$

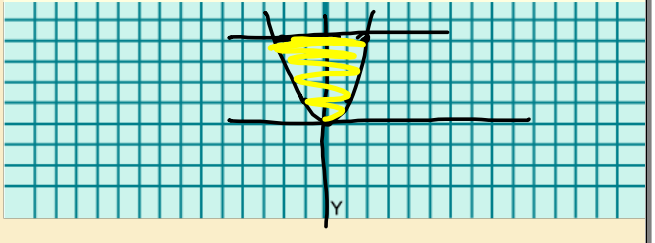
$-2(x-3)-2 = -2x+6-2$
$-2x+4$



$$\int_{-2}^{0} x^2\,dx + \int_{0}^{2}(-2x+4)\,dx$$
$$-\int_{2}^{3}(-2x+4) - \int_{3}^{4}(2x-8)\,dx$$

$y=4 \quad y=x^2$

$$\int_{-2}^{2}(4-x^2)\,dx$$



Volume

Disk $\pi \int_a^b (r_o^2 - r_i^2)\,dx$

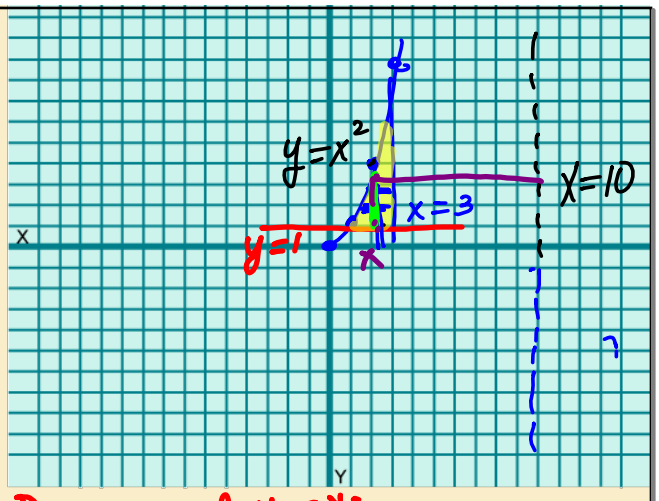
[] is $\perp$ to axis of rev.

Shell $2\pi \int_a^b r(f-g)\,dx$

[] is $\parallel$ to axis of rev.

[] is vertical $y = x$'s

[] is horiz. $x = y$'s

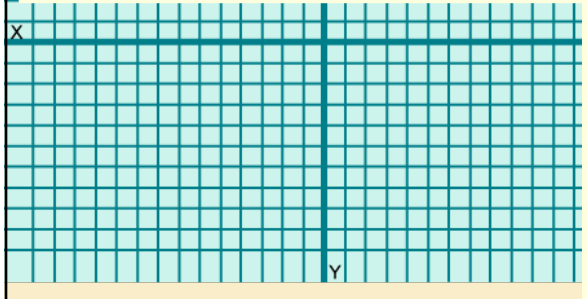


Rev. around x-axis

Disk: $\pi \int_1^3 \left[(x^2)^2 - (1)^2\right] dx$

Around $x = 10$ $x = \sqrt{y}$

$\pi \int_1^9 \left[(10-\sqrt{y})^2 - (10-3)^2\right] dy$



Shell: around x-axis

$2\pi \int_1^9 y\,(3-\sqrt{y})\,dy$

Around $x = 10$ $2\pi \int_1^3 (10-x)(x^2-1)\,dx$

$y = x^2$

Slicing: squares

$A = s^2$ $\int_1^3 (x^2-1)^2\,dx$

$\triangle$ $A = \frac{\sqrt{3}}{4}s^2$

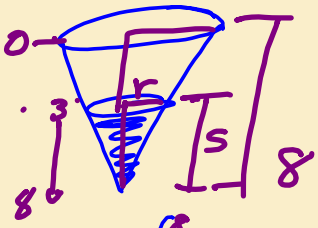
$\cap$ $A = \frac{\pi}{2}r^2$ $\frac{\pi}{2}\int_1^3 \left(\frac{x^2-1}{2}\right)^2$

Work

Springs

$F(x) = Kx$ ← Amt of stretch

$\int Kx$



$W = \int_3^8 \rho \, A(x) \, \text{depth} \, dx$

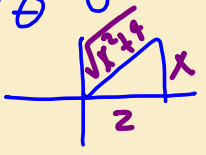
$A(x) = \pi r^2$

Semester 2 Review Day 2

(a) $\int \frac{1}{(4+x^2)^2}\,dx$

$x = 2\tan\theta \rightarrow \frac{x}{2} = \tan\theta$, $\theta = \tan^{-1}\left(\frac{x}{2}\right)$

$dx = 2\sec^2\theta\,d\theta$



$\int \frac{1}{(4+4\tan^2\theta)^2} \cdot 2\sec^2\theta\,d\theta$

$\frac{2}{16}\int \frac{\sec^2\theta}{(1+\tan^2\theta)^2}\,d\theta$ ($1+\tan^2\theta = \sec^2\theta$)

$\frac{1}{8}\int \frac{\sec^2\theta}{\sec^4\theta}\,d\theta$

$\frac{1}{8}\int \frac{1}{\sec^2\theta}\,d\theta$

$\frac{1}{8}\int \cos^2\theta\,d\theta$

$\frac{1}{8}\int \frac{1}{2}(1+\cos 2\theta)\,d\theta$

$\frac{1}{16}\left[\theta + \frac{1}{2}\sin 2\theta\right] + C$ ($\frac{1}{2}\sin 2\theta = \sin\theta\cos\theta$)

$\frac{1}{16}\left[\theta + \sin\theta\cos\theta\right] + C$

$\frac{1}{16}\left[\tan^{-1}\left(\frac{x}{2}\right) + \left(\frac{x}{\sqrt{x^2+4}}\right)\left(\frac{2}{\sqrt{x^2+4}}\right)\right]$

$\frac{1}{16}\left[\tan^{-1}\left(\frac{x}{2}\right) + \frac{1}{2}\cdot\frac{2x}{x^2+4}\right] + C$

$\frac{1}{16}\tan^{-1}\left(\frac{x}{2}\right) + \frac{x}{8(x^2+4)} + C$

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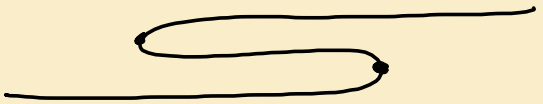

$v = 0$

c) $t = 0.3$

$s(t) = 2t^3 - 9t^2 + 12$

$t =$	0	3	5
$s =$	12	−15	37

27 52 = 79