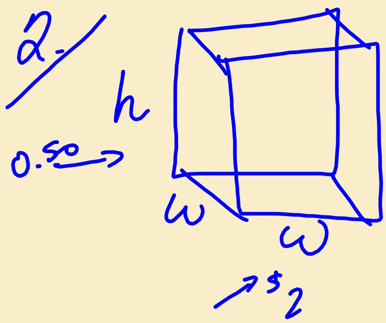
CALCULUS SEMESTER 2 REVIEW

Optimization

2.



w

$(0, \infty)$

$$\lim_{w\to 0} 2w^2 + \frac{256}{w} = 0 + \frac{256}{0} = \infty$$

$$\lim_{w\to \infty} 2w^2 + \frac{256}{w} = \infty$$

$$C(4) = 32 + 64 = \$96$$

$$C = 2w^2 + 0.50 \cdot 4 \cdot wh$$

$$w^2 h = 128 \Leftarrow$$

$$h = \frac{128}{w^2}$$

$$C = 2w^2 + 2w\left(\frac{128}{w^2}\right)$$

$$C = 2w^2 + \frac{256}{w} w^{-1}$$

$$C' = 4w - \frac{256}{w^2} = 0$$

$$w^2 \cdot 4w = \frac{256}{\cancel{w^2}} \cdot \cancel{w^2}$$

$$4w^3 = 256$$

$$\sqrt[3]{w^3} = \sqrt[3]{64}$$

$$w = 4$$

$$h = \frac{128}{4^2} = 8$$

$$\boxed{4m \times 4m \times 8m}$$

April 28, 2022

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Hyperbolic Functions

$$\frac{d}{dx}\cosh x = \sinh x$$

$$\frac{d}{dx}\operatorname{sech} x = -\operatorname{sech} x \tanh x$$

$$f(x) = \sinh(e^{3x})\operatorname{sech}(x^3)$$ Product Rule

$$f'(x) = \sinh(e^{3x}) \cdot -\operatorname{sech}(x^3)\tanh(x^3)\cdot 3x^2 + \operatorname{sech}(x^3)\cdot\cosh(e^{3x})\cdot e^{3x}\cdot 3$$

$$\int x\tanh^5(3x^2)\operatorname{sech}^2(3x^2)\,dx$$

$u = \tanh(3x^2)$

$du = \operatorname{sech}^2(3x^2)\cdot 6x\,dx$

$$\int \cancel{x}\cdot u^5 \cancel{\operatorname{sech}^2(3x^2)}\cdot \frac{du}{6\cancel{x}\cancel{\operatorname{sech}^2(3x^2)}} \qquad \frac{du}{6x\operatorname{sech}^2(3x^2)} = dx$$

$$\frac{1}{6}\int u^5\,du$$

$$\frac{1}{6}\cdot\frac{u^6}{6}$$

$$\frac{1}{36}\tanh^6(3x^2) + C$$

List:

A derivative represents. . .

Integration represents

Give:

$$W = \int \rho \; A(x) \cdot \text{depth} \, dx$$

$$\text{Fluid Force} = \int \rho \; l(x) \, h(x) \, dx$$

Know:

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

Integration

$\int \sin x \, dx = -\cos x + C$

$\int \cos x \, dx = \sin x + C$

$\int \sec^2 x \, dx = \tan x + c$

$\int \csc^2 x \, dx = -\cot x + c$

$\int \sec x \tan dx = \sec x + C$

$\int \csc x \cot x \, dx = -\csc x + C$

$\int e^x dx = e^x + C$

$\int a^x dx = \frac{1}{\ln a} \cdot a^x + C$

$\int \frac{1}{x} dx = \ln|x| + c$

$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C$

$\int \frac{1}{x^2+1} dx = \tan^{-1} x + C$

$\int \frac{1}{|x|\sqrt{x^2-1}} dx = \sec^{-1} x + C$

Methods:

1) Power Rule
2) U-Subst.
3) Inv Trig Func.
4) Integ by parts
5) Trig Integrals

$\int \sin 4x \cos 8x \, dx$ – Sum & prod. Identity

$\int \sin^3 x \cos^4 x \, dx$ – split off $\sin x$
– Use $\sin^2 x + \cos^2 x = 1$
– U-sub

$\int \sin^2 x \cos^4 x \, dx$
– Rewrite as $(\ ^2)^-$
– Use double angle Identities

6) Trig Substitution
7) Partial Fractions

$$\int x(4+2x)^8\,dx \qquad u = 4+2x \Longrightarrow \frac{u-4}{2} = \frac{\cancel{2}x}{\cancel{2}}$$

$$du = 2\,dx$$

$$\int x \cdot u^8 \cdot \frac{du}{2}$$

$$\frac{1}{2}\int \frac{u-4}{2} \cdot u^8 \cdot du$$

$$\frac{1}{4}\int (u^9 - 4u^8)\,du$$

$$\frac{1}{4}\left[\frac{u^{10}}{10} - \frac{4u^9}{9}\right] + C$$

$$\frac{1}{40}(4+2x)^{10} - \frac{1}{9}(4+2x)^9 + C$$

$$\frac{d}{dx}\int_{x^2}^{x^3} \frac{(5-t^3)^7}{\sin t}\,dt$$

$$= \frac{(5-(x^3)^3)^7}{\sin x^3} \cdot 3x^2 - \frac{(5-(x^2)^3)^7}{\sin x^2} \cdot 2x$$