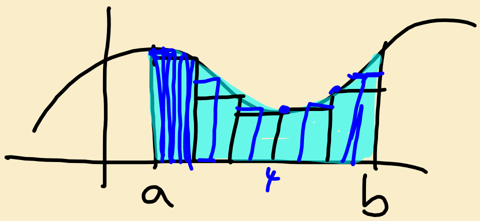
Calculus Review



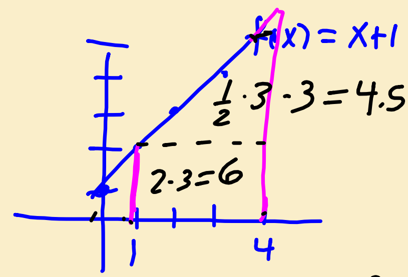
$A = l \cdot w$

$= f(x) \cdot \Delta x$

sum of rectangles

$$\lim_{\Delta x \to 0} \sum_{x=a}^{b} f(x)\,\Delta x = \int_a^b f(x)\,dx$$

Integration represents the area between a curve and an axis.



Area = 10.5 units^2

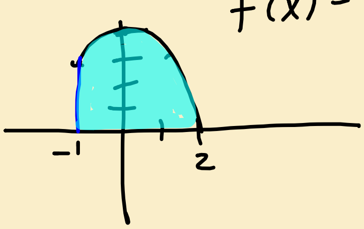
$$\int_1^4 (x+1)\,dx$$

$$= \frac{x^2}{2} + x \Big|_1^4$$

$$= 8 + 4 - \left(\frac{1}{2} + 1\right)$$

$$12 - 1\tfrac{1}{2} = 10.5$$

$f(x) = 4 - x^2$



$$\int_{-1}^{2} (4 - x^2)\,dx$$

$$\int \left(x^3 - 2x^2 + 4\right) dx$$

$$= \frac{x^4}{4} - \frac{2x^3}{3} + 4x + C$$

Find $f'(x)$.

$$f(x) = x^3 - 2x^2 + 4$$

$$f(x) = 3x^2 - 4x$$

Find $f'(x)$

$$f(x) = \left(x^5 - 4x^3 + 7\right)\left(x^7 - 4x^9\right)$$

Product Rule

1st · d'2nd + 2nd · d'1st

$$f'(x) = \underbrace{\left(x^5 - 4x^3 + 7\right)}_{1st}\underbrace{\left(7x^6 - 36x^8\right)}_{d'2nd} + \underbrace{\left(x^7 - 4x^9\right)}_{2nd}\underbrace{\left(5x^4 - 12x^2\right)}_{d'1st}$$

$$f(x) = \frac{7x^{-2} + 3x^5}{4x^9 - 8x^{14}}$$

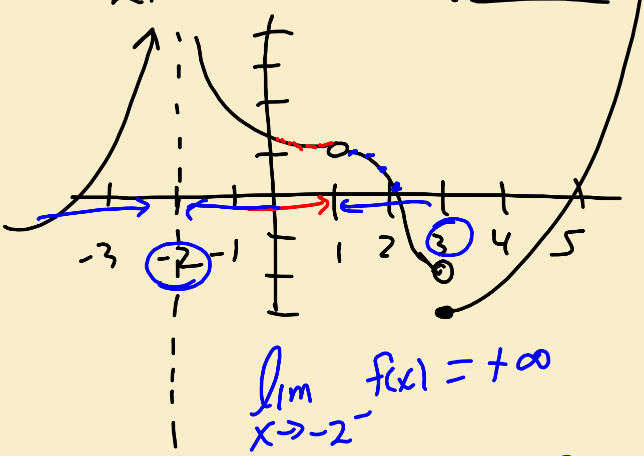
$$\frac{\text{low} \cdot \text{d'high} - \text{high} \cdot \text{d'low}}{\text{low}^2}$$

$$f'(x) = \frac{\left(4x^9 - 8x^{14}\right)\left(-14x^{-3} + 15x^4\right) - \left(7x^{-2} + 3x^5\right)\left(36x^8 - 112x^{13}\right)}{\left(4x^9 - 8x^{14}\right)^2}$$

$$f(x) = \sqrt{x^7 - 8x^4} = \left(x^7 - 8x^4\right)^{1/2}$$

$$f'(x) = \frac{1}{2}\left(x^7 - 8x^4\right)^{-1/2} \cdot \left(7x^6 - 32x^3\right)$$

Limits – Find y-coord!



$\lim_{x \to -2^-} f(x) = +\infty$

$\lim_{x \to -2^+} f(x) = +\infty$

$\lim_{x \to -2^+} f(x) = +\infty$

$\lim_{x \to 1^-} f(x) = 1$

$\lim_{x \to 1^+} f(x) = 1$

$\lim_{x \to 1} f(x) = 1$

$f(1) =$ undef

Find y.

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$$\lim_{x \to 7} \frac{x^2 - 49}{x - 7} = \frac{0}{0}$$

$$= \lim_{x \to 7} \frac{(x+7)\cancel{(x-7)}}{\cancel{x-7}}$$

$$= 7 + 7 = 14$$

1) Sub # in

2) If $\frac{0}{0}$:

try factoring or conjugate

$$\lim_{x \to 5} \frac{\sqrt{x+4} - 3}{x - 5} = \frac{\sqrt{9} - 3}{5 - 5} = \frac{0}{0}$$

$$\lim_{x \to 5} \frac{\sqrt{x+4} - 3}{x - 5} \frac{(\sqrt{x+4} + 3)}{\sqrt{x+4} + 3}$$

$$\lim_{x \to 5} \frac{\cancel{\cancel{x+4-9}}^{\cancel{x-5}}}{\cancel{(x-5)}(\sqrt{x+4} + 3)}$$

$$= \frac{1}{\sqrt{5+4} + 3} = \frac{1}{6}$$

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Find $f'(a)$ given $f(x) = x^3 - 4x + 7$ using the definition of the derivative.

$f'(x) = 3x^2 - 4$

$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$\lim_{x \to a} \frac{x^3 - 4x + \cancel{7} - (a^3 + 4a \cancel{- 7})}{x - a}$$

$$\lim_{x \to a} \frac{(x^3 - a^3)(-4x + 4a)}{x - a}$$

$$\lim_{x \to a} \frac{\cancel{(x - a)}(x^2 + ax + a^2) - 4\cancel{(x - a)}}{\cancel{x - a}}$$

$$= a^2 + a^2 + a^2 - 4$$

$$= 3a^2 - 4$$