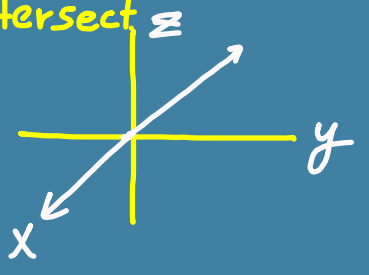
3-Variable Systems

Where 3 planes intersect



Elimination

-3 -4

$x + 8y + 2z = -12$

$3x + y + 7z = 38$

$4x - 3y + 6z = 47$

① $-3x - 24y - 6z = 36$

$3x + y + 7z = 38$

$-23y + z = 74$

② $-4x - 32y - 8z = 48$

$4x - 3y + 6z = 47$

$-35y - 2z = 95$

③ 2

$-23y + z = 74$

$-35y - 2z = 95$

$-46y + 2z = 148$

$-35y - 2z = 95$

$\frac{-81y}{-81} = \frac{243}{-81}$

$y = -3$

④ Sub in $y = -3$

$-23(-3) + z = 74$

$69 + z = 74$

$z = 5$

⑤ $x + 8(-3) + 2(5) = -12$

$x - 24 + 10 = -12$

$x - 14 = -12$

$x = 2$

$(2, -3, 5)$

3-Variable Elimination

1) Group 2 of the equations & eliminate a variable.
2) Group a different pair of equations & eliminate the same variable.
3) Group the 2 resulting equations from steps 1+2 & eliminate a variable.
4) Sub answer in a 2-variable eq. to find a 2nd variable.
5) Sub both answers in a 3-variable eq. & solve.

CRAMER'S RULE

$$4x + 2y - z = 15$$
$$2x - y + 5z = 9$$
$$3x + 2y - z = 12$$

$$x = \frac{\begin{vmatrix} c_1 & y_1 & z_1 \\ c_2 & y_2 & z_2 \\ c_3 & y_3 & z_3 \end{vmatrix}}{\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}} \qquad y = \frac{\begin{vmatrix} x_1 & c_1 & z_1 \\ x_2 & c_2 & z_2 \\ x_3 & c_3 & z_3 \end{vmatrix}}{\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}}$$

$$z = \frac{\begin{vmatrix} 4 & 2 & 15 \\ 2 & -1 & 9 \\ 3 & 2 & 12 \end{vmatrix}}{\begin{vmatrix} 4 & 2 & -1 \\ 2 & -1 & 5 \\ 3 & 2 & -1 \end{vmatrix}} =$$

-3
-9
-17
-14

$$4\begin{vmatrix} -1 & 9 \\ 2 & 12 \end{vmatrix} - 2\begin{vmatrix} 2 & 9 \\ 3 & 12 \end{vmatrix} + 15\begin{vmatrix} 2 & -1 \\ 3 & 2 \end{vmatrix}$$

$$4(-12-18) - 2(24-27) + 15(4+3)$$
$$-30 \qquad -3 \qquad 7$$

$$-120 + 6 + 105 = \boxed{-9}$$

$$4\begin{vmatrix} -1 & 5 \\ 2 & -1 \end{vmatrix} - 2\begin{vmatrix} 2 & 5 \\ 3 & -1 \end{vmatrix} + -1\begin{vmatrix} 2 & -1 \\ 3 & 2 \end{vmatrix}$$

$$4(1-10) - 2(-2-15) + -1(4+3)$$
$$-9 \qquad -17 \qquad 7$$

$$-36 + 34 + -7 = \boxed{-9}$$

$$\boxed{z = \frac{-9}{-9} = 1}$$

Calculator

$$\frac{\det\left(\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}\right)}{\det\left(\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}\right)}$$

x = # of sofas
y = # of chairs
z = # of love seats

$$x + 2y = 1000$$
$$x + z = 1200$$