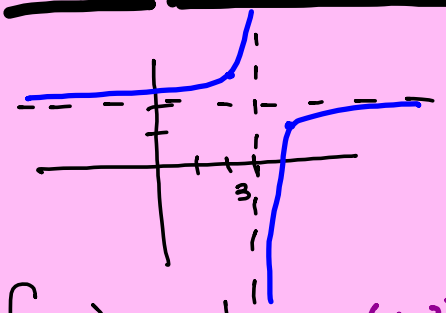
Asymptotes & Continuity



$$f(x) = \frac{-1}{x-3} + \frac{2}{1}\cdot\frac{(x-3)}{(x-3)}$$

$$= \frac{-1+2x-6}{x-3}$$

$$= \frac{2x-7}{x-3}$$

Vertical

$\lim_{x \to \#} f(x) = \pm\infty$ ← where denom = 0

$$\lim_{x\to 3} \frac{2x-7}{x-3} = \frac{-1}{0}$$

Horizontal

$$\lim_{x\to\pm\infty} \frac{2x-7}{x-3} = \#$$

$$f(x) = \frac{x^2 - 2x}{x^2 + x - 6} = \frac{x(x-2)}{(x+3)(x-2)}$$

Hole at $x = 2$

Vertical

$$\lim_{x \to -3} \frac{x}{x+3} = \frac{-3}{0}$$

$$\lim_{x \to -3^-} \frac{x}{x+3} = \frac{-}{-} = +\infty$$

Vertical: $\boxed{x = -3}$

Horizontal

$$\lim_{x \to \infty} \frac{x}{x} = 1$$

$\boxed{y = 1}$

$$f(x) = \frac{\sqrt{36x^2+11}}{3x-5}$$

Vertical

$$\lim_{x \to 5/3^+} \frac{\sqrt{36x^2+11}}{3x-5} = \frac{\sqrt{111}}{0}$$

$$= \frac{+}{+} = +\infty$$

$x = 5/3$

Horizontal

$$\lim_{x \to \infty} \frac{\sqrt{36x^2}}{3x} = \lim_{x \to \infty} \frac{6|x|}{3x}$$

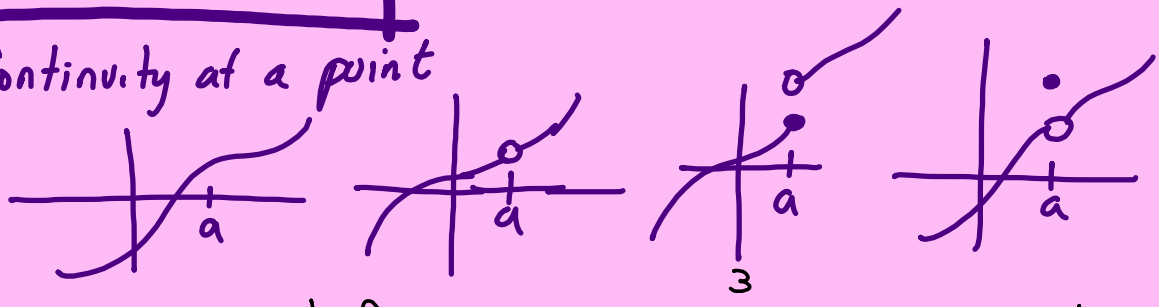
$$= \lim_{x \to \infty} \frac{6x}{3x} = 2$$

$$\lim_{x \to -\infty} \frac{-6x}{3x} = -2$$

$y = 2$ + $y = -2$

CONTINUITY – smooth + unbroken

Continuity at a point



1) $f(a)$ is defined.
2) $\lim_{x \to a} f(x)$ exists.
3) $f(a) = \lim_{x \to a} f(x)$

$$f(x) = \begin{cases} 3x+2 & x < 1 \\ 7-2x^2 & x \geq 1 \end{cases}$$

$a = 1$

1) $f(1) = 7 - 2(1)^2 = 5$
2) $\lim_{x \to 1^-} 3x+2 = 5$

$\lim_{x \to 1^+} 7-2x^2 = 5$

$\lim_{x \to 1} f(x) = 5$

3) $f(1) = \lim_{x \to 1} f(x)$

Continuous! (1)

$$f(x) = \begin{cases} 3x+8 & x < -3 \\ 4 & x = -3 \\ x^2 + 10 & x > -3 \end{cases} \qquad a = -3$$

1) $f(-3) = 4$

2) $\lim_{x \to -3^-} 3x + 8 = -1$

$\lim_{x \to -3^+} x^2 + 10 = -1$

$\lim_{x \to -3} f(x) = -1$

3) $f(-3) \neq \lim_{x \to -3} f(x)$

not continuous

$$f(x) = \sqrt{x^2 - 4x - 21}$$

$(x-7)(x+3)$

$+ \; | \; - \; | \; +$

$-3 \quad 0 \quad 7$

$(-\infty, -3]$ C

$(7, \infty)$ C

$[-\infty, 7]$ D

$(-3, 7)$ D