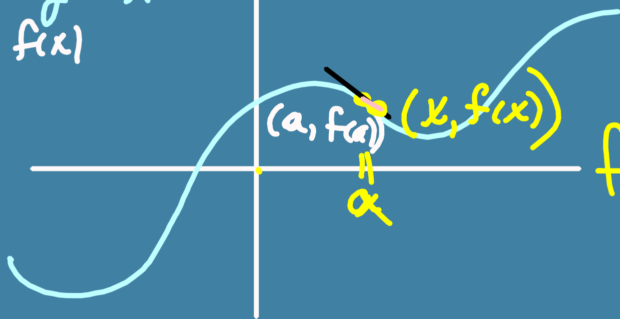
DERIVATIVES

– represents the slope of a line tangent to a curve

$y = x^4 - 3x^2 + 7x + 1$



1st Def. of Deriv.

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

↗ lets x approach a to make a tangent line

↖ slope

func	1st Deriv	2nd Deriv.
$f(x)$	$f'(x)$	$f''(x)$
$y =$	$\frac{dy}{dx} = \frac{\Delta y}{\Delta x}$	$\frac{d^2y}{dx^2}$
$y =$	y'	y''
$y =$	$D_x y$	

Isaac Newton

Gottfried von Leibniz

$$f'(x) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$$

$f(x) = x^3-2x^2+x-1$ Find $f'(a)$.

$$\lim_{x\to a} \frac{x^3-2x^2+x-\cancel{1}+(-a^3+2a^2-a+\cancel{1})}{x-a} = \frac{0}{0}$$

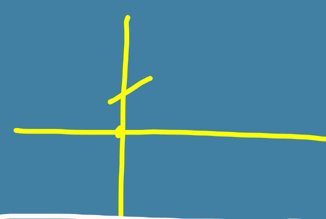
$$\lim_{x\to a} \frac{(x^3-a^3)+\overset{-2(x^2-a^2)}{(-2x^2+2a^2)}+(x-a)}{\cancel{x}-a}$$

$$\lim_{x\to a} \frac{\cancel{(x-a)}(x^2+ax+a^2)-2(x+a)\cancel{(x-a)}+\cancel{(x-a)}}{\cancel{x-a}}$$

$$\lim_{x\to a} x^2+ax+a^2-2(x+a)+1$$

$$= a^2+a^2+a^2-2(2a)+1$$

$$= \boxed{3a^2-4a+1} = 0$$



$f(x) = x^3-2x^2+x^1-1$.

$f'(x) = 3x^2-4x^1+1$

Power Rule

$$\frac{d}{dx}x^n = n\cdot x^{n-1}$$

Find eq. of the tangent at $x=-1$.

$m = f'(-1) = 3(-1)^2-4(-1)+1$

$\boxed{m=8}$

$x=-1$ $f(-1) = (-1)^3-2(-1)^2+-1-1$

$= -1-2-1-1$

$= -5$

$\boxed{(-1,-5)}$

Point-Slope

$y-y_1 = m(x-x_1)$

$y- -5 = 8(x- -1)$

$y+5 = 8x+8$

$\boxed{y = 8x+3}$

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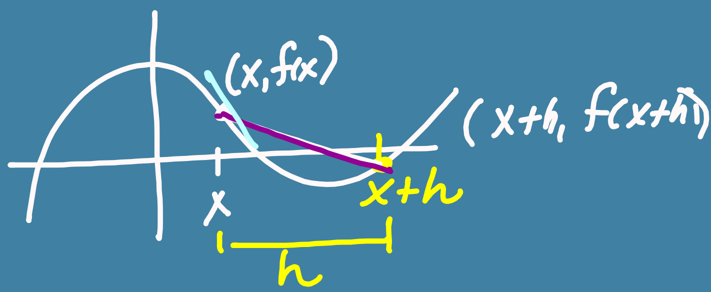

2nd Def. of Deriv

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{x+h-x} = \boxed{\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}}$$

$f(x) = x^2 + 4x + 1$ Find $f'(x)$ using

$2x + 4$

$$\lim_{h\to 0} \frac{(x+h)^2 + 4(x+h) + 1 - (x^2+4x+1)}{h}$$

$$\lim_{h\to 0} \frac{\cancel{x^2}+2xh+h^2+\cancel{4x}+4h+\cancel{1}-\cancel{x^2}-\cancel{4x}-\cancel{1}}{h}$$

$$\lim_{h\to 0} \frac{2xh+h^2+4h}{h}$$

$$\lim_{h\to 0} \frac{\cancel{h}(2x+h+4)}{\cancel{h}} = 2x+4$$

$f(x) = \sin x$

$$\boxed{\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

$$\lim_{h \to 0} \frac{\sin(x+h) - \sin x}{h}$$

$$\lim_{h \to 0} \frac{\sin x \cosh + \cos x \sinh - \sin x}{h}$$

$$\lim_{x \to 0} \frac{\sin x}{x}$$

$$\lim_{x \to 0} \frac{1 - \cos x}{x}$$

$$\lim_{h \to 0} \frac{\sin x \cosh - \sin x}{h} + \frac{\cos x \sinh}{h}$$

$$\lim_{h \to 0} -\sin x \left(\frac{1 - \cosh}{h}\right) + \cos x \cdot \frac{\sinh}{h}$$

$$= -\sin x \cdot 0 + \cos x \cdot 1$$

$$= \cos x$$

$\frac{d}{dx}\sin x = \cos x$ $\frac{d}{dx}\cos x = -\sin x$

$\frac{d}{dx}\tan x = \sec^2 x$ $\frac{d}{dx}\cot x = -\csc^2 x$

$\frac{d}{dx}\sec x = \sec x \tan x$ $\frac{d}{dx}\csc x = -\csc x \cot x$

$f(x) = 4\tan x + 3\csc x$

$f'(x) = 4\sec^2 x - 3\csc x\cot x$

$f(x) = \sin x(\csc x + 1)$

$= \sin x \csc x + \sin x$

$= \cancel{\sin x} \cdot \frac{1}{\cancel{\sin x}} + \sin x$

$f'(x) = 0 + \cos x$

$f(x) = 3x^8 - \frac{1}{3x^5} - 7\sqrt[3]{x^2} + 31$

$= 3x^8 - \frac{1}{3}x^{-5} - 7x^{2/3} + 31$

$f'(x) = 24x^7 + \frac{5}{3}x^{-6} - \frac{14}{3}x^{-1/3}$

$= 24x^7 + \frac{5}{3x^6} - \frac{14}{3x^{1/3}}$