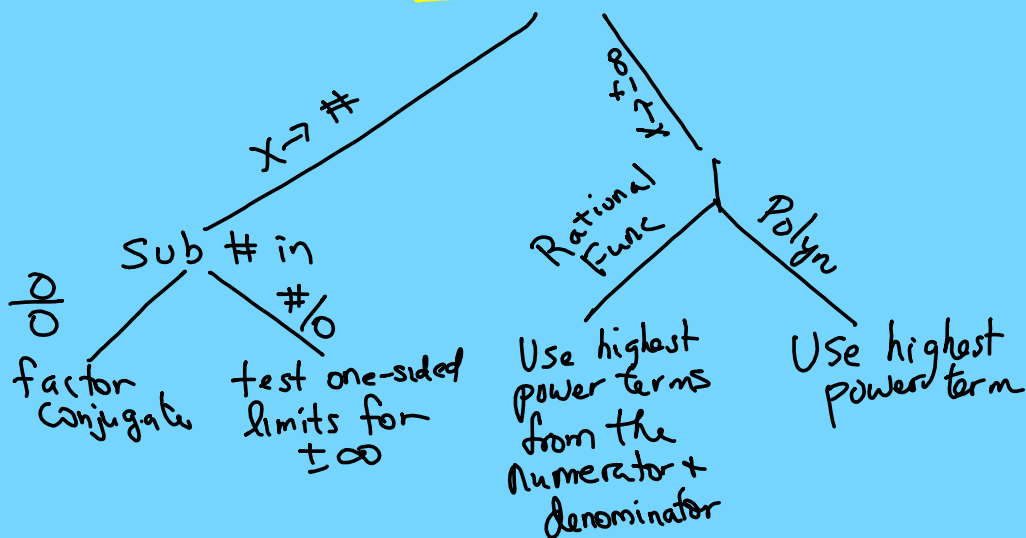


LIMITS



One-Sided Limits

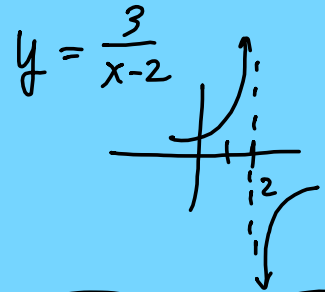
$$\lim_{x \rightarrow 5^-} \frac{2x+3}{x-1} = \frac{13}{4}$$

$$\lim_{x \rightarrow 2} \frac{3x}{x-2} = \frac{6}{0} = \boxed{\text{DNE}}$$

$$\lim_{x \rightarrow 2^-} \frac{3x}{x-2} = \frac{+}{-} = -\infty$$

$$\lim_{x \rightarrow 2^+} \frac{3x}{x-2} = \frac{+}{+} = +\infty$$

$$\lim_{x \rightarrow -3^+} \frac{8x}{(x+3)^2} = \frac{-24}{0} = \frac{-}{+} = \boxed{-\infty}$$

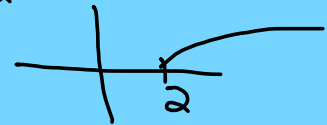


$$\lim_{x \rightarrow 4^+} \frac{8x^2}{(x-4)^4} = \frac{128}{0} = \boxed{+\infty}$$

$$\lim_{x \rightarrow 4^-} \frac{8x^2}{(x-4)^4} = \frac{+}{+} = +\infty$$

$$\lim_{x \rightarrow 4^+} \frac{8x^2}{(x-4)^4} = \frac{+}{+} = +\infty$$

$$\lim_{x \rightarrow 2^-} \sqrt{x-2} = \text{DNE}$$



$$f(x) = \begin{cases} \frac{2x+1}{3x-1} & x < -1 \\ \frac{1}{(x-1)^2} & x > -1 \end{cases}$$

$$\lim_{x \rightarrow -1} f(x) = \left(\frac{1}{4} \right)$$

$$\lim_{x \rightarrow -1^-} \frac{2x+1}{3x-1} = \frac{-2+1}{-3-1} = \frac{-1}{-4} = \frac{1}{4}$$

$$\lim_{x \rightarrow -1^+} \frac{1}{(x-1)^2} = \frac{1}{(-2)^2} = \frac{1}{4}$$

$$\lim_{x \rightarrow 3} \frac{1}{(x-1)^2} = \frac{1}{4}$$

$$f(x) = \begin{cases} x^2-3 & x < 2 \\ 4x+7 & x = 2 \\ \frac{2}{x-2} & x > 2 \end{cases}$$

$$\lim_{x \rightarrow 2} f(x) = \text{DNE}$$

$$\lim_{x \rightarrow 2^-} x^2-3 = 2^2-3 = 1$$

$$\lim_{x \rightarrow 2^+} \frac{2}{x-2} = \frac{2}{2-2} = \frac{2}{0} = \frac{+}{-} = -\infty$$

2.1

LIMITS to $\pm\infty$ rigorous

$$\lim_{x \rightarrow \infty} \frac{\frac{1}{x^2} x^2}{\frac{1}{x^2} 7x^2 + 3} = \frac{\infty^2}{7\infty^2 + 3}$$

$$\lim_{x \rightarrow \infty} \frac{x^2}{7x^2} = \frac{1}{7}$$

$$\lim_{x \rightarrow \infty} \frac{\frac{x^2}{x^2}}{\frac{7x^2}{x^2} + \frac{3}{x^2}}$$

$$\lim_{x \rightarrow \infty} \frac{1}{7 + \frac{3}{x^2}} = \frac{1}{7 + \frac{3}{\infty} \rightarrow 0} = \boxed{\frac{1}{7}}$$

$$\lim_{x \rightarrow -\infty} \frac{4x^3 - 3x^2 + 1}{5x^4 + 2x^2 - 9}$$

$$\lim_{x \rightarrow -\infty} \frac{4x^3}{5x^4} = \lim_{x \rightarrow -\infty} \frac{4}{5x} = \frac{4}{-\infty} = \textcircled{0}$$

$$\lim_{y \rightarrow -\infty} \frac{5y^3 + 4}{3y + 7} = \lim_{y \rightarrow -\infty} \frac{5y^3}{3y} = \lim_{y \rightarrow -\infty} \frac{5}{3}y^2 = \frac{5}{3}(-\infty)^2 = \infty$$

$$\lim_{x \rightarrow -\infty} \frac{\sqrt{x^2 + 13}}{6x + 5} = \lim_{x \rightarrow -\infty} \frac{\sqrt[2]{x^2}}{6x} = \lim_{x \rightarrow -\infty} \frac{|x|}{6x} = \lim_{x \rightarrow -\infty} \frac{-x}{6x} = -\frac{1}{6}$$

even-even-odd

$$\lim_{z \rightarrow -\infty} -3z^2 - 7z = \lim_{z \rightarrow -\infty} -3z^2 = -3(-\infty)^2 = -3 \cdot +\infty = -\infty$$