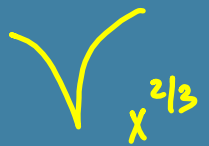
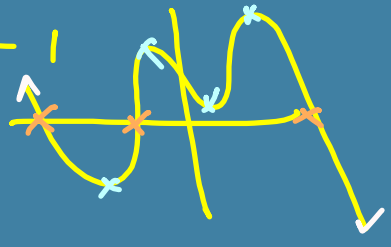
Polynomial & Rational Functions

Polynomials

1) one or more terms
2) whole # exponents

$\frac{1}{2}x^4 + 3x^3 - 2x^2 + 7x - 1$

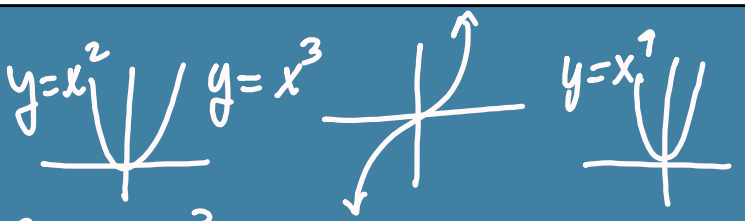




Graphs – rollercoaster graphs

1) smooth, rounded turns
2) **continuous** (no end pts, no holes, no asymptotes, no sharp pts)

End Behavior



Degree of a polyn. = highest power

$f(x) = -2x^3 - 2x^2 + 4x - 5$

X-Intercepts = degree = zeros = roots

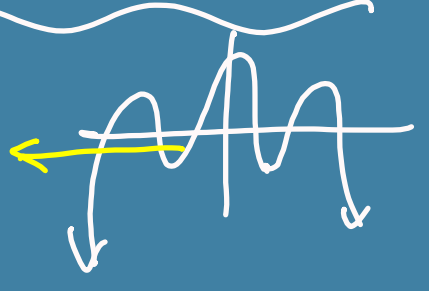
Negative leading coeff. = ends go in opposite directions

Odd power — ends go opposite direction

Even power — ends go in same direction

max # of relative max/mins = Degree − 1

$f(x) = -3x^6 + 7x^5 - 28x^3 + 2x^2 - 9$



# of Zeros = 6

# of Rel Extrema = 5

$\lim_{x \to -\infty} f(x) = -\infty$ ↑ y

far to the left

$\lim_{x \to \infty} f(x) = -\infty$

far to the right

$f(x) = 3x^5 + 7x^3 - 9x + 1$

Zeros = 5

Rel Extrema = 4

$\lim_{x \to -\infty} f(x) = -\infty$ (left) ↑ y

$\lim_{x \to \infty} f(x) = +\infty$ (Right) ↑ y

Solving Polynomials

$(2x^3 - 8x^2 + 3x - 12) = 0$

$2x^2(x-4) + 3(x-4) = 0$

$(x-4)(2x^2+3) = 0$

$x = 4$

$2x^2 + 3 = 0$

$\frac{2x^2}{2} = \frac{-3}{2}$

$\sqrt{x^2} = \sqrt{-3/2}$

$x = \frac{i\sqrt{3}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$= \pm \frac{i\sqrt{6}}{2}$

$x^2 - 3x - 4 = 0$

Solve.

$2x^5 + 3x^4 - x^3 + 9x^2 - 55x - 30 = 0$

$(2x\_)(x-2)(x\_)(x\_)(x\_)$

$\pm1\ \pm2\ \pm3\ \pm5\ \pm6\ \pm10\ \pm15\ \pm30$ / ±2

$x = \#$

2 | 2 3 −1 9 −55 −30

+ | 0 4 14 26 70 30

2 7 13 35 15 0

$(x-2)(2x^4+7x^3+13x^2+35x+15)$

$\pm1\ \pm3\ \pm5\ \pm15$ / ±2

$2x^4+7x^3\ldots\ldots+15 \mid x=-3$ 0

−3 | 2 7 13 35 15

+ −6 −3 −30 −15

2 1 10 5 0

$(x-2)(x+3)((2x^3+x^2)+(10x+5))$

$(x-2)(x+3)[x^2(2x+1)+5(2x+1)]$

$(x-2)(x+3)(2x+1)(x^2+5)$

$\sqrt{x^2=-5}$

$=\pm i\sqrt{5}$

$x = 2, -3, -\frac{1}{2}, \pm i\sqrt{5}$