

DERIVATIVE RULES

PRODUCT RULE

$$f(x) = 3x^2 \cdot 4x^5$$

~~$$f'(x) = 6x \cdot 20x^5$$~~

$$= 120x^6$$

$$f(x) = 12x^7$$

$$f'(x) = 84x^6$$

$$f'(x) = 3x^2 \cdot 20x^4 + 4x^5 \cdot 6x$$

$$= 60x^6 + 24x^6$$

$$= 84x^6$$

$$\frac{d}{dx} f \cdot g = f \cdot g' + g \cdot f'$$

$$= \text{1st} \cdot \text{d'2nd} + \text{2nd} \cdot \text{d'1st}$$

$$f(x) = (x^6 - 3x^8 + 7)(3x^{-4} + 2x^{1/3} - 5)$$

$$f'(x) = (x^6 - 3x^8 + 7)\left(-12x^{-5} + \frac{2}{3}x^{-4/3}\right) + (3x^{-4} + 2x^{1/3} - 5)(6x^5 - 24x^7)$$

QUOTIENT RULE

$$f(x) = \frac{1}{x} = x^{-1}$$

$$f'(x) = -x^{-2} = \frac{-1}{x^2}$$

$$\frac{d}{dx} \frac{f}{g} = \frac{g \cdot f' - f \cdot g'}{g^2} = \frac{\text{low} \cdot \text{d'high} - \text{high} \cdot \text{d'low}}{\text{low}^2}$$

$$f(x) = \frac{4 \sin x - 3x^5}{8x^{2/9} - \csc x}$$

$$f'(x) = \frac{(8x^{2/9} - \csc x)(4 \cos x - 15x^4) - (4 \sin x - 3x^5)\left(\frac{16}{9}x^{-7/9} + \csc x \cot x\right)}{(8x^{2/9} - \csc x)^2}$$

CHAIN RULE

$$\frac{d}{dx} f[g(h(x))] = f'[g(h(x))] \cdot g'(h(x)) \cdot h'(x)$$

$$f(x) = (7x^9 - 3x^5)^8$$

$$f'(x) = 8(7x^9 - 3x^5)^7 \cdot (63x^8 - 15x^4)$$

$$f(x) = \sqrt[4]{(x^7 - 5x^3)(x^4 + 9x^7)} = \left[(x^7 - 5x^3)(x^4 + 9x^7) \right]^{1/4}$$

$$f'(x) = \frac{1}{4} \left[(x^7 - 5x^3)(x^4 + 9x^7) \right]^{-3/4} \cdot \left[(x^7 - 5x^3)(4x^3 + 63x^6) + (x^4 + 9x^7)(7x^6 - 15x^2) \right]$$

$$f(x) = \left[\frac{\tan x \cos x}{(x^2+3)^9 (x^{11}-2x^5)^4} \right]^{19}$$

$$f'(x) = 19 \left[\frac{\tan x \cos x}{(x^2+3)^9 (x^{11}-2x^5)^4} \right]^{18} \cdot \left[\frac{(x^2+3)^9 (x^{11}-2x^5)^4}{(x^2+3)^9 (x^{11}-2x^5)^4} \right] \cdot \left[\tan x \cdot (-\sin x) + \cos x \cdot \sec^2 x \right]$$

$$- \tan x \cos x \cdot \left[\frac{1}{(x^2+3)^9} \cdot 4(x^{11}-2x^5)^3 \cdot (11x^{10} - 10x^4) + \frac{(x^{11}-2x^5)^4}{(x^2+3)^8} \cdot 7x^6 \right]$$

$$\left[(x^2+3)^9 (x^{11}-2x^5)^4 \right]^2$$

Annotations:

- Green bracket above $(x^2+3)^9 (x^{11}-2x^5)^4$ labeled "low".
- Green bracket above $\left[\frac{(x^2+3)^9 (x^{11}-2x^5)^4}{(x^2+3)^9 (x^{11}-2x^5)^4} \right]$ labeled "d'high".
- Green bracket above $(x^2+3)^9$ labeled "high".
- Purple bracket above $(x^2+3)^9$ labeled "1st".
- Purple bracket above $4(x^{11}-2x^5)^3$ labeled "d'2nd".
- Purple bracket above $(x^{11}-2x^5)^4$ labeled "d'low 2nd".
- Purple bracket above $(x^2+3)^8$ labeled "d'1st".