Limits of Special Functions

LIMITS

Special Limits

Think about graph

$x \to \#$

Sub # in

$\frac{0}{0}$

factor
conjugate

$\#/0$

test one-sided limits for $\pm\infty$

$x \to \pm\infty$

Rational

Use highest power terms from num & denom

Polyn

Use highest power term

Special Limits

$$\lim_{x\to 0} \frac{\sin(nx)}{nx} = 1$$

$$\lim_{x\to 0} \frac{1-\cos(nx)}{nx} = 0$$

$$\lim_{x\to 0} \frac{5\cdot\sin(5x)}{5\cdot x}$$

$$5\lim_{x\to 0} \frac{\sin 5x}{5x}$$

$$5\cdot 1 = 5$$

$$\lim_{x\to 0} \frac{3(1-\cos(12x))}{3\cdot 4x}$$

$$3\lim_{x\to 0} \frac{1-\cos(12x)}{12x}$$

$$3\cdot 0 = 0$$

$$\lim_{x\to 0} \frac{\frac{1}{x}\sin 6x}{\frac{1}{x}\sin 8x}$$

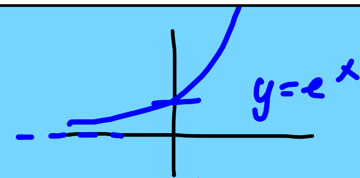
$$\lim_{x\to 0} \frac{\frac{6}{6}\cdot\frac{\sin 6x}{x}}{\frac{8}{8}\cdot\frac{\sin 8x}{x}}$$

$$\frac{6}{8}\lim_{x\to 0} \frac{\frac{\sin 6x}{6x}}{\frac{\sin 8x}{8x}}$$

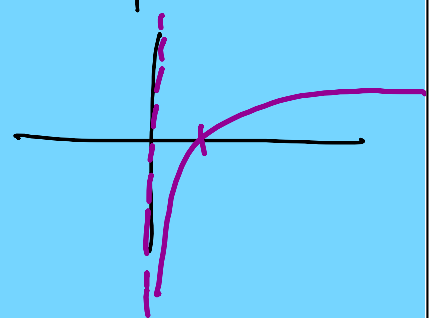
$$= \frac{3}{4}\cdot\frac{1}{1}$$

$$= 3/4$$

$\lim\limits_{x\to\infty} e^x = +\infty$ $\qquad$ $\lim\limits_{x\to-\infty} e^x = 0$



$\lim\limits_{x\to\infty} \ln x = +\infty$ $\qquad$ $\lim\limits_{x\to 0^+} \ln x = -\infty$



$\lim\limits_{x\to 0^+} e^{1/x} =$

$\lim\limits_{x\to 0^+} \frac{1}{x} = \frac{1}{0} = \frac{+}{+} = +\infty$

$e^{\infty} = +\infty$

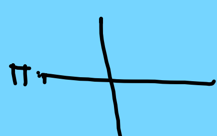
$\lim\limits_{x\to 0^-} \frac{1}{x} = \frac{+}{-} = -\infty$

$e^{-\infty} = 0$

$$\lim_{x\to-\infty}\frac{1-e^{x}}{e^{-x}} = \frac{1-e^{-\infty}}{e^{+\infty}} = \frac{1-0}{+\infty} = \frac{1}{\infty} = 0$$

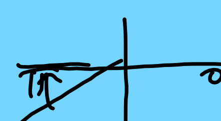
$$\lim_{x\to\pi^{+}} \sec x = \sec \pi$$
$$= -1$$

$\cos 0 = 1$
$\sec 0 = 1$

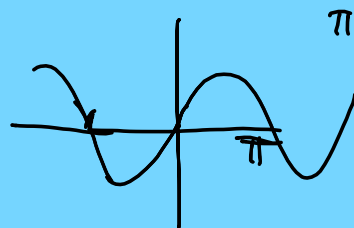


$\sin \pi/6 = \frac{1}{2}$
$\sin^{-1} 1/2 = \pi/6$

$$\lim_{x\to\frac{1}{2}^{+}}\frac{\cos^{-1}x}{\sin 2\pi x} = \frac{\cos^{-1} 1/2}{\sin(\cancel{2}\pi\cdot\frac{1}{\cancel{2}})} = \frac{\pi/3}{0}$$



π^{+}



$$= \frac{+}{-} = -\infty$$