October 14, 2022

IMAGINARY + COMPLEX NUMBERS

→ square roots of negative numbers $i = \sqrt{-1}$

$i^1 = i$

$i^2 = i \cdot i = \sqrt{-1} \cdot \sqrt{-1} = -1$

$i^3 = i^2 \cdot i^1 = -1 \cdot i = -i$

$i^4 = i^2 \cdot i^2 = -1 \cdot -1 = 1$

$i^5 = \cancel{i}^4 \cdot i^1 = 1 \cdot i = i$

$i^6 = \cancel{i}^4 \cdot i^2$

.... $i = i$ ___.25

.... $i^2 = -1$ ___.5

.... $i^3 = -i$ ___.75

... $i^4 = 1$ ___.0

I won! I won!
(with 2 negatives
in the middle)

$\frac{15}{4} = 3.\boxed{75}$

3/4

Use the remainder →

$$\begin{array}{r} 3 \\ 4\overline{)15} \\ \underline{12} \\ \boxed{3} \end{array}$$

$i^{15} = i^3 = -i$

$i^{482} = i^2 = -1$

$\frac{482}{4} = 120.5$

$3i^{234} - 2i^{65} + i^{10,203}$

$\frac{234}{4} = 58.5$ i^2 $\frac{65}{4} = 16.25$ i^1 $\frac{10,203}{4} = 255.75$ i^3

$= 3(-1) - 2(i^1) + -i$

$= -3 - 2i + -i$

$= \boxed{-3 - 3i}$

$\sqrt{-6}\cdot\sqrt{\overset{16\cdot 2}{-32}}$

$= i\sqrt{6}\cdot 4i\sqrt{2}$

$= 4i^2\sqrt{\underset{4\cdot 3}{12}}$

$= 4(-1)2\sqrt{3}$

$= -8\sqrt{3}$

Solve.

$3x^2+65=11$

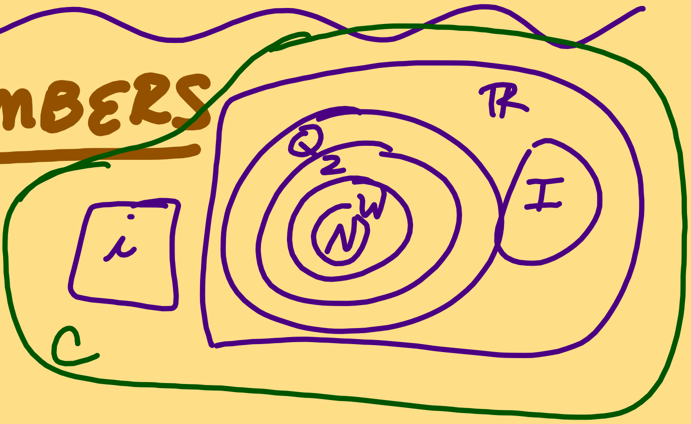
$\quad -65 \quad -65$

$\frac{3x^2}{3}=\frac{-54}{3}$

$\sqrt{x^2}=\sqrt{\underset{-1\cdot 9\cdot 2}{-18}}$

$x=\pm 3i\sqrt{2}$

COMPLEX NUMBERS



→ have 2 parts
1 real, 1 imag

$\underset{\substack{\uparrow\\ \text{real}}}{a} + \underset{\substack{\uparrow\\ \text{imag}}}{bi}$

$4+2i$

$8+0i$

$0-7i$

COMPLEX ARITHMETIC

$(5+3i)+(7-8i)=12-5i$

$(9-6i)+(+3\overset{-}{+}4i)=12-10i$

$(6-8i)(5+2i)$ FOIL!

$= 30+12i-40i+16\cancel{i^2}$

$= 46-28i$

$(2-7i)^2=(2-7i)(2-7i)$

$= 4-14i-14i\overset{-}{+}49\cancel{i^2}$

$= -45-28i$

$$\frac{6}{7i}\cdot\frac{i}{i} = \frac{6i}{-7\cancel{i^2}} = -\frac{6i}{7}$$

$$\frac{6}{7\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}$$

$$\frac{3+2i}{5i}\cdot\frac{i}{i} = \frac{3i\overset{-}{+}2\cancel{i^2}}{-5\cancel{i^2}}$$

$$= \frac{-3i+2}{+5}$$

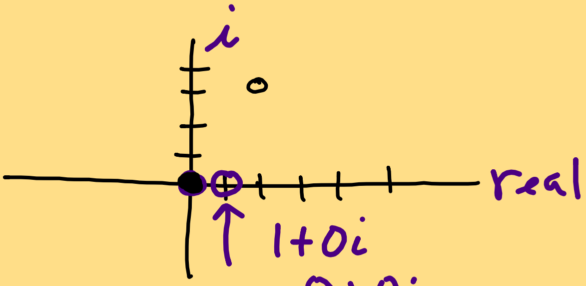
$$= \frac{2-3i}{5}$$

$$\frac{4+2i}{3+5i}\cdot\frac{(3-5i)}{(3-5i)}$$

$$\frac{12-20i+6i+10\cancel{i^2}}{9+25\cancel{i^2}} = \frac{22-14i}{34} = \boxed{\frac{11-7i}{17}}$$

$$\overline{3+5\sqrt{2}}\,(3-5\sqrt{2})$$

FRACTALS – 1980 – Benoit Mandelbrot



0+0i

2+3i

$f(x) = x^2 + c$

$f(x) = x^2 + 0 + 0i$

Iteratin

$f(0) = 0^2 + 0 + 0i = 0$

$f(0) = 0^2 + 0 + 0i = 0$

Fractals – self-similar

$f(x) = x^2 - 10$

$f(3) = 3^2 - 10$

$f(x) = x^2 + 1 + 0i$

$f(0) = 0^2 + 1 + 0i = 1$

$f(1) = 1^2 + 1 + 0i = 2$

$f(2) = 2^2 + (1 + 0i) = 5$

$f(5) = 5^2 + (1 + 0i) = 26$