a) $y + \ln(xy) = 1$

$$\frac{dy}{dx} + \frac{1}{xy} \cdot \left[x \cdot \frac{dy}{dx} + y \cdot 1\right] = 0$$

$$\frac{dy}{dx} + \frac{1}{y}\frac{dy}{dx} + \frac{1}{x} = 0$$

$$\frac{dy}{dx}\left(\frac{y}{y}1 + \frac{1}{y}\right) = -\frac{1}{x}$$

$$\frac{dy}{dx}\left(\frac{y+1}{\cancel{y}}\right) = -\frac{1}{x} \cdot \frac{y}{y+1}$$

$$\frac{dy}{dx} = \frac{-y}{x(y+1)}$$

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$$f(x) = (\sin x)^{\ln x} \qquad a = \frac{\pi}{2}$$

$$f(x) = e^{\ln(\sin x)^{\ln x}}$$

$$f(x) = e^{\ln x \cdot \ln(\sin x)}$$

$$f'(x) = e^{\ln x \cdot \ln(\sin x)} \cdot \left[\ln x \cdot \frac{1}{\sin x} \cdot \cos x + \ln(\sin x) \cdot \frac{1}{x}\right]$$

$$= \sin x^{\ln x} \cdot \left[\frac{x \cdot \ln x \cot x}{x \cdot 1} + \frac{\ln(\sin x)}{x}\right]$$

$$= (\sin x)^{\ln x} \left[\frac{x \ln x \cot x + \ln(\sin x)}{x}\right]$$

$$= 1^{\cancel{\ln \pi/2}} \quad \frac{\cancel{\frac{\pi}{2}\ln\left(\frac{\pi}{2}\right)\cdot 0} + \ln(1)}{\pi/2} = \frac{0}{\pi/2} = 0$$

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Inverse Trig Functions

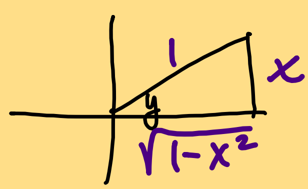
$y = \sin^{-1} x$

$\frac{x}{1} = \sin y$

$1 = \cos y \frac{dy}{dx}$

$\frac{1}{\cos y} = \frac{dy}{dx}$

$\frac{1}{\sqrt{1-x^2}} = \frac{dy}{dx}$



$a^2 + x^2 = 1$

$\sqrt{a^2} = \sqrt{1-x^2}$

$\frac{dy}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$ $\qquad$ $\frac{dy}{dx} \cos^{-1} = \frac{-1}{\sqrt{1-x^2}}$

$\frac{dy}{dx} \tan^{-1} x = \frac{1}{x^2+1}$ $\qquad$ $\frac{d}{dx} \cot^{-1} x = \frac{-1}{x^2+1}$

$\frac{d}{dx} \sec^{-1} x = \frac{1}{|x|\sqrt{x^2-1}}$ $\qquad$ $\frac{d}{dx} \csc^{-1} x = \frac{-1}{|x|\sqrt{x^2-1}}$

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$$f(x) = \sin^{-1}(7x^5)$$

$$f'(x) = \frac{1}{\sqrt{1-(7x^5)^2}} \cdot 35x^4$$

$$= \frac{35x^4}{\sqrt{1-49x^{10}}}$$

$$f(x) = \left[\csc^{-1}(x^4)\right]\left[\tan^{-1}(\ln x^2)\right]$$

$$f'(x) = \csc^{-1}(x^4) \cdot \frac{1}{(\ln x^2)^2+1} \cdot \frac{1}{x^{\cancel{2}}} \cdot 2\cancel{x} + \tan^{-1}(\ln x^2) \cdot \frac{-1}{|x^4|\sqrt{x^8-1}} \cdot 4x^3$$

$$= \frac{2\csc^{-1}(x^4)}{x\left[(\ln x^2)^2+1\right]} - \frac{4x^3\tan^{-1}(\ln x^2)}{x^4\sqrt{x^8-1}}$$

L'Hopital's Rule

$$\lim_{x \to 2} \frac{x^2-4}{x-2} = \frac{0}{0}$$

$$\lim_{x \to 2} \frac{(x+2)\cancel{(x-2)}}{\cancel{x-2}} = 4$$

$$\lim_{x \to 2} \frac{2x}{1} = 4$$

Indeterminate Forms

$\frac{0}{0}$, $\frac{\infty}{\infty}$

$0 \cdot \infty$

L'Hopital's Rule - must $\frac{0}{0}$, $\frac{\infty}{\infty}$

$$\lim_{x \to \#} \frac{f(x)}{g(x)} = \lim_{x \to \#} \frac{f'(x)}{g'(x)}$$

$$\lim_{x\to 1} \frac{x^3-3x^2+5x-3}{x^2+x-2} = \frac{0}{0}$$

$$\lim_{x\to 1} \frac{3x^2-6x+5}{2x+1} = \frac{3-6+5}{2+1} = \frac{2}{3}$$

$$\lim_{x\to 0} \frac{e^x-1-x}{\cos(2x)-1} = \frac{1-1-0}{1-1} = \frac{0}{0}$$

$$\lim_{x\to 0} \frac{e^x-1}{-\sin(2x)\cdot 2} = \frac{1-1}{0\cdot 2} = \frac{0}{0}$$

$$\lim_{x\to 0} \frac{e^x}{-2\cos(2x)\cdot 2} = \frac{1}{-2\cdot 1\cdot 2} = \frac{1}{-4}$$

$$\lim_{x \to 0^+} \frac{1 - \ln x}{e^{1/x}} \qquad x^{-1}$$

$$= \frac{1 + {+\infty}}{e^{\cancel{1/0}\,+\infty}} = \frac{\infty}{\infty}$$

$$\lim_{x \to -\infty} e^x = 0 \qquad \lim_{x \to \infty} e^x = \infty$$

$$\lim_{x \to 0^+} \ln x = -\infty \qquad \lim_{x \to \infty} \ln x = \infty$$

$$\lim_{x \to 0^+} \frac{-\frac{1}{x}}{e^{1/x} \cdot -1x^{-2}}$$

$$\lim_{x \to 0^+} \frac{-\frac{1}{x}}{\frac{-e^{1/x}}{x^2}}$$

$$\lim_{x \to 0^+} +\frac{1}{\cancel{x}} \cdot + \frac{x^{\cancel{2}}}{e^{1/x}}$$

$$\lim_{x \to 0^+} \frac{x}{e^{1/x}} = \frac{0}{e^{\cancel{1/0}\,\infty}} = \frac{0}{\infty} = 0$$