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DERIVATIVES OF EXPONENTIAL & LOG FUNCTIONS

$f(x) = e^x$ $\qquad \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ $\qquad \frac{d}{dx} e^x = e^x$

$$\lim_{h \to 0} \frac{e^{x+h} - e^x}{h}$$

$$\lim_{h \to 0} \frac{e^x \cdot e^h - e^x}{h}$$

$$\lim_{h \to 0} \frac{e^x (e^h - 1)}{h}$$

$$\lim_{h \to 0} e^x \left(\frac{e^h - 1}{h}\right) = e^x \cdot 1$$
$$= e^x$$

$$f(x) = e^{x^2+4x}$$

$$f'(x) = e^{x^2+4x} \cdot (2x+4)$$

$$f(x) = x^2 \cdot e^{4x^3}$$

$$f'(x) = \underbrace{x^2 \cdot e^{4x^3} \cdot 12x^2} + \underbrace{e^{4x^3} \cdot 2x}$$
$$= 2xe^{4x^3}\left[6x^3 + 1\right]$$

$e^y = e^{\ln x}$

$y = \ln_e x$ Find $\frac{dy}{dx}$.

$e \quad e$

$e^y = x$

$e^y \cdot \frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{e^y}$

$= \frac{1}{x}$

$y = \ln x \quad \frac{dy}{dx} = \frac{1}{x}$

$y = \ln(x^3 - 3x^5)$

$y' = \frac{1}{x^3 - 3x^5} \cdot (3x^2 - 15x^4)$

$= \frac{3x^2 - 15x^4}{x^3 - 3x^5}$

$= \frac{\cancel{x^2}(3 - 15x^2)}{\cancel{x^2}(x - 3x^3)}$

$y = a^{x}$ $\qquad y = 2^{x}$ $\qquad y = 5^{x}$ $\qquad \frac{d}{dx} a^{x} = \ln a \cdot a^{x}$

$\ln y = x \cdot \ln a$ (3x)

$\frac{1}{y}\frac{dy}{dx} = \ln a$

$\frac{dy}{dx} = y \cdot \ln a$

$= a^{x} \cdot \ln a$

$y = 7^{\sin x}$

$y' = \ln 7 \cdot 7^{\sin x} \cdot \cos x$

$f(x) = 7^{\frac{e^{x^2}}{\tan x}}$

$f'(x) = \ln 7 \cdot 7^{\frac{e^{x^2}}{\tan x}} \cdot \left[\frac{\tan x \cdot e^{x^2} \cdot 2x - e^{x^2} \cdot \sec^2 x}{\tan^2 x}\right]$

$= \ln 7 \cdot 7^{\frac{e^{x^2}}{\tan x}} \cdot e^{x^2}\left[\frac{2x\tan x - \sec^2 x}{\tan^2 x}\right]$

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$$f(x) = \log_8 3x^7$$

$$= \frac{\ln 3x^7}{\ln 8}$$

$$= \frac{1}{\ln 8} \ln 3x^7$$

$$= \frac{1}{\ln 8} \cdot \frac{1}{\cancel{3}\cancel{x^7}^{\,x}} \cdot \cancel{21}^{\,7}\cancel{x^6}$$

$$= \frac{7}{\ln 8} \cdot \frac{1}{x} \text{ OR } \frac{7}{x \ln 8}$$

Change of Base Formula

$$\log_b a = \frac{\ln a}{\ln b}$$

Variable raised to variable exponent. (Tower Functions)

1) Exponentiate using $e^{\ln}$

2) plug the exponent in front of $\ln$

3) Do deriv. with product rule

4) simplify.

$$f(x) = x^{x^2}$$

$$f(x) = e^{\ln x^{x^2}}$$

$$f(x) = e^{x^2 \cdot \ln x}$$

$$f'(x) = e^{x^2 \cdot \ln x} \cdot \left[\cancel{x^2} \cdot \frac{1}{\cancel{x}} + \ln x \cdot 2x \right]$$

$$= x^{x^2} \left[x + 2x \ln x \right]$$

$$= x^{x^2} \cdot x^1 \left[1 + 2 \ln x \right]$$

$$= x^{x^2+1} \left[1 + 2 \ln x \right]$$

$$f(x) = x^{\sin x} = e^{\ln x^{\sin x}} = e^{\sin x \cdot \ln x}$$

$$f'(x) = e^{\sin x \cdot \ln x} \cdot \left[\sin x \cdot \frac{1}{x} + \ln x \cdot \cos x \right]$$

$$= x^{\sin x} \left[\frac{\sin x}{x} + \frac{x \cdot \ln x \cos x}{x \cdot 1} \right]$$

$$= x^{\sin x} \left[\frac{\sin x + x \ln x \cos x}{x} \right]$$

$$= \frac{x^{\sin x}}{x^1} \left[\sin x + x \ln x \cos x \right]$$

$$= x^{\sin x - 1} \left[\sin x + x \ln x \cos x \right]$$

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Additional Examples

Change log x to ln x.

1/ln 3 is a constant number. Leave out front and do product rule on remainder.

$$f(x) = x^2 \cdot \log_3 x^7 = x^2 \cdot \frac{\ln x^7}{\ln 3} = \frac{1}{\ln 3} \cdot \underbrace{x^2 \cdot \ln x^7}$$

$$f'(x) = \frac{1}{\ln 3}\left[x^{\cancel{2}} \cdot \frac{1}{\cancel{x^7}} \cdot 7\cancel{x^6} + \ln x^7 \cdot 2x\right]$$

$$= \frac{1}{\ln 3}\left[7x + 2x \ln x^7\right]$$

$$= \frac{x}{\ln 3}\left[7 + 2\ln x^7\right]$$

$$f(x) = \frac{7^{x^2} \sec\left(e^{5x^7}\right)}{\ln\left(8^{\sin x}\right)}$$

$$f'(x) = \frac{\ln\left(8^{\sin x}\right) \cdot \left[7^{x^2} \cdot \sec\left(e^{5x^7}\right)\tan\left(e^{5x^7}\right) \cdot e^{5x^7} \cdot 35x^6 + \sec\left(e^{5x^7}\right) \cdot \ln 7 \cdot 7^{x^2} \cdot 2x\right] - 7^{x^2}\sec\left(e^{5x^7}\right) \cdot \frac{1}{8^{\sin x}} \cdot \ln 8 \cdot 8^{\sin x} \cdot \cos x}{\left[\ln\left(8^{\sin x}\right)\right]^2}$$

Additional Examples

$$f(x) = \frac{7^{x^2} \sec\left(e^{5x^7}\right)}{\ln\left(8^{\sin x}\right)}$$

$$f'(x) = \frac{\ln\left(8^{\sin x}\right) \cdot \left[7^{x^2} \cdot \sec\left(e^{5x^7}\right)\tan\left(e^{5x^7}\right) \cdot e^{5x^7} \cdot 35x^6 + \sec\left(e^{5x^7}\right) \cdot \ln 7 \cdot 7^{x^2} \cdot 2x\right] - 7^{x^2} \sec\left(e^{5x^7}\right) \cdot \frac{1}{8^{\sin x}} \cdot \ln 8 \cdot 8^{\sin x} \cdot \cos x}{\left[\ln\left(8^{\sin x}\right)\right]^2}$$