More L'Hopital's Rule

$$\lim_{x\to 0^+} x^2 \cdot \ln x = 0 \cdot -\infty$$

$$\lim_{x\to 0^+} \frac{\ln x}{x^{-2}} \cdot \frac{-\infty}{+\infty}$$

$$\lim_{x\to 0^+} \frac{\frac{1}{x}}{\frac{-2}{x^3}}$$

$$\lim_{x\to 0^+} \frac{1}{x} \cdot \frac{x^3}{-2} = \frac{0}{-2} = 0$$

Indeterminate forms

$\frac{0}{0}, \frac{\infty}{\infty}$

$0 \cdot \infty, \ \infty - \infty$

$0^0, \ 1^\infty, \ \infty^0$

must rearrange into fraction form of $\frac{0}{0}$ or $\frac{\infty}{\infty}$

$$\lim_{x\to 0^+}\left(\csc x-\frac{1}{x}\right)=\infty-\infty$$

$$\lim_{x\to 0^+}\left(\frac{1}{\sin x}-\frac{1}{x}\right)\cdot\frac{\sin x}{\sin x}\cdot\frac{x}{x}$$

$$\lim_{x\to 0^+}\frac{x-\sin x}{x\sin x}=\frac{0-0}{0\cdot 0}$$

$$\lim_{x\to 0^+}\frac{1-\cos x}{x\cdot\cos x+\sin x\cdot 1}=\frac{1-1}{0\cdot 1+0\cdot 1}=\frac{0}{0}$$

$$\lim_{x\to 0^+}\frac{\sin x}{x\cdot-\sin x+\cos x\cdot 1+\cos x}$$

$$\lim_{x\to 0^+}\frac{\sin x}{-x\sin x+2\cos x}=\frac{0}{0\cdot 0+2\cdot 1}=\frac{0}{2}=$$

$$\boxed{0}$$

$$\lim_{x\to\infty} x^{1/x} = \infty^{\frac{1}{\infty}} = \infty^0$$

$\ln x^{1/x}$

$$\lim_{x\to\infty} e^{\ln x^{1/x}}$$

$$\lim_{x\to\infty} e^{\frac{1}{x}\ln x}$$

$$\lim_{x\to\infty} \frac{\ln x}{x} = \frac{\infty}{\infty}$$

$$\lim_{x\to\infty} \frac{\frac{1}{x}}{1} = \frac{1}{\infty} = 0$$

$$= e^0 = \boxed{1}$$

Steps.

$\ln x^{f(x)}$

1) Rewrite as $e^{\ln x^{f(x)}} = e^{f(x)\cdot \ln x}$
2) Rearrange exponent to fraction form as $\frac{0}{0}$ or $\frac{\infty}{\infty}$
3) Perform L'Hopital's Rule
4) Write answer as $e^{\#}$

$\ln(1-x^2)$

$-$ ~~(natural)~~ $-$

$-1 \qquad 1$

$(-1, 1)$

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = \left(1+\frac{1}{\infty}\right)^{\infty} = 1^{\infty}$$

$$\lim_{x\to\infty} e^{x\ln\left(1+\frac{1}{x}\right)}$$

$$\lim_{x\to\infty}\frac{\ln\left(1+\frac{1}{x}\right)}{x^{-1}} = \frac{\ln\left(1+\frac{1}{\infty}\right)}{\frac{1}{\infty}} = \frac{0}{0}$$

$$\lim_{x\to\infty}\frac{\frac{1}{\left(1+\frac{1}{x}\right)}\cdot\cancel{-1x^{-2}}}{\cancel{-x^{-2}}}$$

$$= \frac{1}{1+\frac{1}{\infty}} = \frac{1}{1+0} = 1$$

$$= e^1 = \boxed{e}$$

Exp + Log Functions Review

Guaranteed on the test:

1c.
3i – quadr formula
3m – Unfoil
Law of Cooling

$$\left(125^{-2/3} + 4^{-1}\right)^{-1}$$

$$\left(\frac{1}{\sqrt[3]{125^2}} + \frac{1}{4}\right)^{-1}$$

$$\left(\frac{1}{5^2} + \frac{1}{4}\right)^{-1}$$

$$\left(\frac{1}{25} + \frac{1}{4}\right)^{-1}$$

$$\left(\frac{1}{100} + \frac{25}{100}\right)^{-1}$$

$$\left(\frac{29}{100}\right)^{-1}$$

$$= \frac{100}{29}$$

Like 1 (d-i)

$\log_4 64 = \log_4 4^3 = 3$

$\log_7 \sqrt[5]{49} = \log_7 \sqrt[5]{7^2} = \log_7 7^{2/5} = 2/5$

$\ln \frac{1}{\sqrt{e^3}} = \ln e^{-3/2} = -3/2$

3) like a

$\log_x 64 = 6$

$x^{\log_x 64} = x^6$

$\sqrt[6]{64} = \sqrt[6]{x^6}$

$\pm 2 = x$

3(d)

$2\log_6 4^2 - \frac{1}{3}\log_6 8^{1/3} = 3\log_6 x$

$\log_6 16 - \log_6 2 = \log_6 x^3$

$\log_6 \frac{16}{2} = \log_6 x^3$

$6^{\log_6 8} = 6^{\log_6 x^3}$

$\sqrt[3]{8} = \sqrt[3]{x^3}$

$2 = x$

Check!

Like 3(i)

$\ln 2x + \ln(x+2) = 3$

$e^{\ln(2x^2+4x)} = e^3$

$2x^2+4x = e^3$

$2x^2+4x-e^3 = 0$

$x = \frac{-4 \pm \sqrt{16-4(2)(-e^3)}}{2(2)}$

$= \frac{-4 \pm \sqrt{16+8e^3}}{4}$

$2.323, \cancel{-4.323}$

3(m)

$e^{2x} + 5e^x = 14$ $\qquad x^2 + 5x = 14$

$e^{2x} + 5e^x - 14 = 0$

$(e^x + 7)(e^x - 2) = 0$

$e^x + 7 = 0$ $\qquad e^x - 2 = 0$

$\ln e^x = \ln(7)$ $\qquad \ln e^x = \ln 2$

$x = \ln 2$

Found old leather moccasins. Carbon-14 half-life 5730 yrs. Has 10% of its Carbon-14. How old are they?

$N = N_0 e^{kt}$

$0.5 = 1\, e^{K \cdot 5730}$

$\ln 0.5 = \ln e^{5730K}$

$\frac{\ln(0.5)}{5730} = \frac{5730K}{5730}$

$-0.000121 = K$

$1.21\ E-4$

$\ln 0.10 = \ln e^{-0.000121t}$

$\frac{\ln(0.1)}{-0.000121} = \frac{-0.000121t}{-0.000121}$

$19{,}030\ \text{yrs} = t$

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