SPECIAL DERIVATIVES

Implicit Differentiation

Explicit

$* y = 3x^2 + 7x - 4$

$\frac{dy}{dx} = 6x + 7$

Implicit

$y^2 + 3xy + 7 = 2 - 5y$

* Cannot express y in terms of x
* multiple variables.

Find $\frac{dy}{dx}$. "normal"

$y^2 + x^3 + y^3 = 5$

$(3x^2+7x-4)^2 + x^3 + (3x^2+7x-4)^3 = 5$

$2(3x^2+7x-4)^1 \cdot (6x+7) + 3x^2 + 3(3x^2+7x-4)^2 \cdot (6x+7) = 0$

$2y^1 \cdot \frac{dy}{dx} + 3x^2 + 3y^2 \cdot \frac{dy}{dx} = 0$

$2y\frac{dy}{dx} + 3x^2 + 3y^2\frac{dy}{dx} = 0$

$2y\frac{dy}{dx} + 3y^2\frac{dy}{dx} = -3x^2$

$\frac{dy}{dx}(2y + 3y^2) = -3x^2$

$\frac{dy}{dx} = \frac{-3x^2}{2y+3y^2}$

Find $\frac{dy}{dx}$.

$$(3x^2)(y^2) + 4y^5 = 6\sin y + 8x^5$$

$$\overbrace{3x^2 \cdot 2y\frac{dy}{dx} + y^2 \cdot 6x}^{\text{product rule}} + 20y^4\frac{dy}{dx} = 6\cos y\frac{dy}{dx} + 40x^4$$

$$6x^2y\frac{dy}{dx} + 6xy^2 + 20y^4\frac{dy}{dx} = 6\cos y\frac{dy}{dx} + 40x^4$$

$$\frac{dy}{dx}\left[6x^2y + 20y^4 - 6\cos y\right] = 40x^4 - 6xy^2$$

$$\frac{dy}{dx} = \frac{40x^4 - 6x^2y}{6x^2y + 20y^4 - 6\cos y} = \frac{20x^4 - 3x^2y}{3x^2y + 10y^4 - 3\cos y}$$

Find the eq. of the tangent line at $(1, 0)$.

$$m = \frac{20(1)^4 - 3(1)^2(0)}{3(1)^2(0) - 10(0)^4 - 3\cos 0} = \frac{20 - 0}{0 - 0 - 3} = \frac{-20}{3}$$

$$y - 0 = -\frac{20}{3}(x - 1)$$

$$\boxed{y = -\frac{20}{3}x + \frac{20}{3}}$$

Find $\frac{da}{dp}$. $\leftarrow$ normal $\quad 3r^7 + 6a^5 - 4p = p^7$

$$21r^6 \frac{dr}{dp} + 30a^4 \frac{da}{dp} - 4 = 7p^6$$

$$30a^4 \frac{da}{dp} = 7p^6 + 4 - 21r^6 \frac{dr}{dp}$$

$$\frac{da}{dp} = \frac{7p^6 + 4 - 21r^6 \frac{dr}{dp}}{30a^4}$$

Find $\frac{dy}{dt}$. $\quad 4x^2 + 2y^5 = \cos x$

$$8x \frac{dx}{dt} + 10y^4 \frac{dy}{dt} = -\sin x \frac{dx}{dt}$$

$$10y^4 \frac{dy}{dt} = (-8x - \sin x) \frac{dx}{dt}$$

$$\frac{dy}{dt} = \frac{(-8x + \sin x) \frac{dx}{dt}}{10y^4}$$