More L'Hopital's Rule

$$\lim_{x \to 0^+} x^2 \cdot \ln x = 0 \cdot -\infty$$

$$\lim_{x \to 0^+} \frac{\ln x}{x^{-2}} = \frac{-\infty}{+\infty}$$

$$\lim_{x \to 0^+} \frac{\frac{1}{x}}{\frac{-2}{x^3}}$$

$$\lim_{x \to 0^+} \frac{1}{\cancel{x}} \cdot \frac{x^{\cancel{3}2}}{-2} = \frac{0}{-2}$$

$$= 0$$

Indeterminate forms

$\frac{0}{0}$, $\frac{\infty}{\infty}$

$0 \cdot \infty$, $\infty - \infty$

0^0, 1^∞, ∞^0

must rearrange into fraction form of $\frac{0}{0}$ or $\frac{\infty}{\infty}$

October 17, 2022

$$\lim_{x\to 0^+}\left(\csc x - \frac{1}{x}\right) = \infty - \infty$$

$$\lim_{x\to 0^+}\left(\frac{x\cdot 1}{x\cdot \sin x} - \frac{1\cdot \sin x}{x\cdot \sin x}\right)$$

$$\lim_{x\to 0^+} \frac{x-\sin x}{x\sin x} = \frac{0-0}{0\cdot 0}$$

$$\lim_{x\to 0^+} \frac{1-\cos x}{\underbrace{x\cdot\cos x}+\sin x\cdot 1} = \frac{1-1}{0\cdot 1+0\cdot 1} = \frac{0}{0}$$

$$\lim_{x\to 0^+} \frac{\sin x}{x\cdot -\sin x+\cos x\cdot 1+\cos x}$$

$$\lim_{x\to 0^+} \frac{\sin x}{-x\sin x+2\cos x} = \frac{0}{0\cdot 0+2\cdot 1} = \frac{0}{2} = \boxed{0}$$

$$\lim_{x\to\infty} x^{1/x} = \infty^{1/\infty} = \infty^{0}$$

$$\lim_{x\to\infty} e^{\ln x^{1/x}}$$

$$\lim_{x\to\infty} e^{\frac{1}{x}\ln x}$$

$$\lim_{x\to\infty} \frac{\ln x}{x} = \frac{\infty}{\infty}$$

$$\lim_{x\to\infty} \frac{\frac{1}{x}}{1} = \frac{1}{\infty} = 0$$

$$= e^{0} = \boxed{1}$$

Steps.

1) Rewrite as $e^{\ln x^{f(x)}} = e^{f(x)\cdot\ln x}$
2) Rearrange exponent to fraction form as $\frac{0}{0}$ or $\frac{\infty}{\infty}$
3) Perform L'Hopital's Rule
4) Write answer as e^{*}

$\ln(1-x^2)$

$-\quad(+)\quad-$

$-1\qquad 1$

$(-1, 1)$

October 17, 2022

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = \left(1+\frac{1}{\infty}\right)^{\infty} = 1^{\infty}$$

$$\lim_{x\to\infty} e^{x\ln\left(1+\frac{1}{x}\right)}$$

$$\lim_{x\to\infty}\frac{\ln\left(1+\frac{1}{x}\right)}{x^{-1}} = \frac{\ln\left(1+\frac{1}{\infty}\right)}{\frac{1}{\infty}} = \frac{0}{0}$$

$$\lim_{x\to\infty}\frac{\frac{1}{\left(1+\frac{1}{x}\right)}\cdot \cancel{-1x^{-2}}}{\cancel{-x^{-2}}}$$

$$= \frac{1}{1+\frac{1}{\infty}} = \frac{1}{1+0} = 1$$

$$= e^1 = \boxed{e}$$

$$\lim_{x\to 0^+} (\csc x)^{\sin x} = \infty^0$$

$$\sin x = \frac{1}{\csc x}$$

$$\lim_{x\to 0^+} e^{\sin x \ln(\csc x)}$$

$$\lim_{x\to 0^+} \frac{\ln(\overset{\infty}{\csc x})}{\csc x} = \frac{\infty}{\infty}$$

$$\lim_{x\to 0^+} \frac{\frac{1}{\csc x}\cdot \cancel{-\csc x \cot x}}{\cancel{-\csc x \cot x}} =$$

$$= \frac{1}{\infty} = 0$$

$$e^0 = 1$$