

## Natural Log Operations

$$\ln x + \ln(x+3) = 2$$

$$e^{\ln(x^2+3x)} = e^2$$

$$x^2 + 3x = e^2$$

$$x^2 + 3x - e^2 = 0$$

$$x = \frac{-3 \pm \sqrt{9 - 4(1)(-e^2)}}{2(1)}$$

$$= \frac{-3 \pm \sqrt{9 + 4e^2}}{2}$$

$$= 1.605, -4.605$$

$$\frac{2e^{2x-5}}{2} = \frac{32}{2}$$

$$\ln e^{2x-5} = \ln 16$$

$$2x-5 = \ln(16)$$

$$\frac{2x}{2} = \frac{\ln(16) + 5}{2}$$

$$x \approx 3.886$$

$$4^{2x+3} = 75$$

$$\ln(4)^{2x+3} = \ln 75$$

$$\frac{(2x+3)\ln 4}{\ln 4} = \frac{\ln 75}{\ln 4}$$

$$2x+3 = \frac{\ln(75)}{\ln(4)}$$

$$\frac{2x}{2} = \frac{\frac{\ln(75)}{\ln(4)} - 3}{2}$$

$$x = 0.057$$

$$e^{2x} + 3e^x = 28$$

$$e^{2x} + 3e^x - 28 = 0$$

$$(e^x + 7)(e^x - 4) = 0$$

$$e^x + 7 = 0 \quad e^x - 4 = 0$$

$$\ln e^x = \ln -7 \quad \ln e^x = \ln 4$$

$$x = \ln(-7)$$

$$x = \ln(4) \approx 1.386$$

Radioactive Iodine has a half-life of 60 days  
 It is considered to be safe when 5% or less  
 is left. How many days will it take to reach  
 a safe level.

$$N = N_0 e^{kt}$$

Find  $k$ .

$$0.5 = 1 e^{k \cdot 60}$$

$$\ln 0.5 = \ln e^{60k}$$

$$\frac{\ln(0.5)}{60} = \frac{60k}{60}$$

$$-0.0116 = k$$

$$\frac{0.05}{1} = \frac{1 \cdot e^{-0.0116t}}{1}$$

$$\ln 0.05 = \ln e^{-0.0116t}$$

$$\frac{\ln(0.05)}{-0.0116} = \frac{-0.0116t}{-0.0116}$$

$$\boxed{259 \text{ days} = t}$$

## Newton's Law of Cooling

$$u = T + (u_0 - T)e^{kt}$$

Find K.

$$72 = 71 + (75 - 71)e^{k \cdot 1}$$

$$72 = 71 + 4e^k$$

$$\frac{1}{4} = \frac{4e^k}{4}$$

$$\ln 0.25 = \ln e^k$$

$$-1.386 = k$$

Bought Elvis' guitar  
for \$1,000,000.

Appreciate 6% per  
year  
When will it be worth  
2,000,000

$$N = N_0(1+r)^t$$

$$\frac{2,000,000}{1,000,000} = \frac{1,000,000(1+0.06)^t}{1,000,000}$$

$$2 = 1.06^t$$

$$\frac{\log 2}{\log 1.06} = \frac{t \cdot \log 1.06}{\log 1.06}$$

Room Temp = 71°

Normal body = 98.6  
°F

Body found = 75°

1 hr. later = 72°

When did the murder  
occur?

$$75 = 71 + (98.6 - 71)e^{-1.386t}$$

$$75 = 71 + 27.6e^{-1.386t}$$

$$4 = \frac{27.6e^{-1.386t}}{27.6}$$

$$\ln\left(\frac{4}{27.6}\right) = \ln e^{-1.386t}$$

$$\frac{\ln\left(\frac{4}{27.6}\right)}{-1.386} = \frac{-1.386t}{-1.386}$$

$$1.39 \text{ hrs} = t$$

$$0.39 \cdot 60 = 23.4$$

1 hr 23 min.

