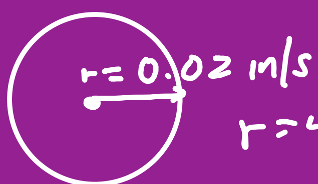
RELATED RATES

$\frac{mi}{h}$ $\frac{m}{s}$ $\frac{cm}{h}$ $\frac{gal}{min}$ $\frac{rad}{sec}$

- rate of one part of the situation impacts the rate of another part.

Example 1



$$\frac{d}{dt}\left[A = \pi r^2\right]$$

$$1 \cdot \frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

$$\frac{dA}{dt} = 2\pi(\underset{in}{4})(\underset{\frac{in}{sec}}{0.02})$$

$$= 0.16\pi \approx \boxed{0.5 \ \frac{in^2}{sec}}$$

1) Draw a picture
2) Label with variables (changing) & constants (not changing)
3) Set up a formula
4) Do derivative with respect to time using implicit differentiation.
5) Identify the rate to be found.
6) Fill in values & solve.

2/

$\frac{dV}{dt} = -0.2\ \frac{m^3}{min}$

Instant when Surface Area $= 0.64\pi\ m^2$

Find: rate of r

$\frac{d}{dt}\left[V = \frac{4}{3}\pi r^3\right]$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\underset{\frac{m^3}{min}\cdot\frac{1}{m^2}}{-0.2} = 4\pi \underset{m^2}{(0.4)^2} \frac{dr}{dt}$

$\frac{-0.2}{0.64\pi} = \frac{0.64\pi}{0.64\pi}\frac{dr}{dt}$

$-0.0994 = \frac{dr}{dt}$

≈ -0.1 m/min

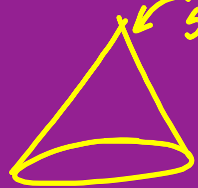
$S.A. = 4\pi r^2$

$\frac{0.64\cancel{\pi}}{4\cancel{\pi}} = \frac{4\pi r^2}{4\pi}$

$\sqrt{0.16} = \sqrt{r^2}$

$0.4 = r$

3)



$5\ ft^3/min$

$h = 2r \qquad r = \frac{h}{2}$

Find rate of height when 10 ft high

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h$$

$$V = \frac{1}{3}\pi \cdot \frac{h^2}{4} \cdot h$$

$$\frac{d}{dt}\left[V = \frac{1}{12}\pi h^3\right]$$

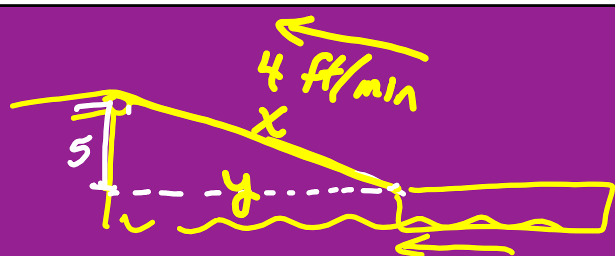
$$\frac{dV}{dt} = \frac{1}{4}\pi h^2 \frac{dh}{dt}$$

$$+5 = \frac{1}{4}\pi(10)^2 \frac{dh}{dt}$$

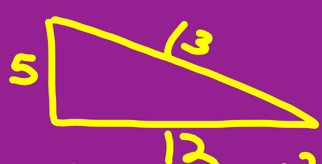
$\frac{ft^3}{min}$ $\qquad ft^2$

$$\frac{5}{25\pi} = \frac{25\pi}{25\pi}\frac{dh}{dt}$$

$$\frac{1}{5\pi}\frac{ft}{min} = \frac{dh}{dt}$$



Find $\frac{dy}{dt}$



$$5^2 + 12^2 = x^2$$

$$5^2 + y^2 = x^2$$

$$\frac{d}{dt}\left[25 + y^2 = x^2\right]$$

$$2y\frac{dy}{dt} = 2x\frac{dx}{dt}$$

$$2(12)\frac{dy}{dt} = 2(13)(-4)$$

$$\frac{24\frac{dy}{dt}}{24} = \frac{-104}{24}$$

$$\frac{dy}{dt} = -\frac{13}{3}\ \frac{\text{ft}}{\text{min}}$$