November 21, 2022

# ABSOLUTE EXTREMA

Back 2 days – Finding relative extrema

**1st deriv.**

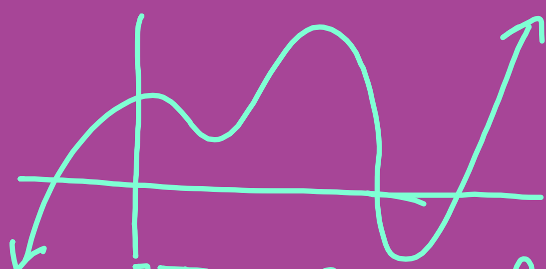
2 5

**2nd deriv**

$f''(\text{crit pt}) = \begin{cases} + \cup & \text{min} \\ - \cap & \text{max} \end{cases}$

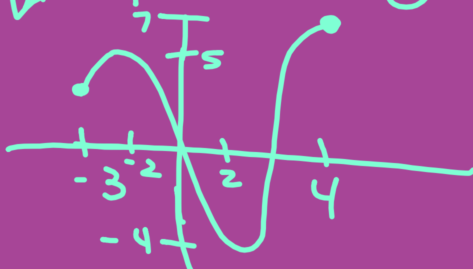
$f''(\text{crit pt}) = 0$ inconclusive
must go back to 1st deriv. test

# ABSOLUTE EXTREMA



$(-\infty, \infty)$

No abs. extrema




Abs max at $(4, 7)$

Abs min at $(2, -4)$

Interval: $[-3, 4]$

$f(x) = x^2 - 3x + 2 \quad [0, 5]$

$f'(x) = 2x - 3 = 0$

$x = 3/2$

| $x$ | |
|---|---|
| 0 | 2 |
| 3/2 | -1/4 |
| 5 | 12 |

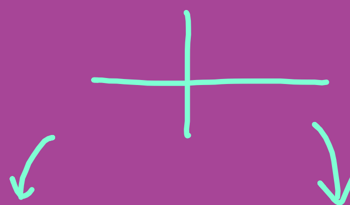
Abs max: (5, 12)
Abs min: (3/2, -1/4)

If given closed intervals (brackets)
1) Find crit pts.
2) Sub crit pts & end points in f to find highest & lowest values.

$f(x) = 3 - 4x - 2x^2 \quad (-\infty, \infty)$

$\lim_{x \to -\infty} -2x^2 = -\infty \quad \leftarrow$ No abs. min.

$\lim_{x \to +\infty} -2x^2 = -\infty$



$f'(x) = -4 - 4x = 0$

$-4 = 4x$

$-1 = x$

| $x$ | $f(x)$ |
|---|---|
| $-1$ | $3+4-2=5$ |

$\boxed{\text{No abs min} \\ \text{Abs max at } (-1, 5)}$

If open interval $(\_, \_)$

1) Check limits of end points.
2) Find crit pts.
3) Find y-coord of crit pts. & compare to limits

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$f(x) = (x^3-1)^{2/3}$ $(-1, 4]$

$\lim_{x \to -1^+} (x^3-1)^{2/3} = (-1-1)^{2/3} = -2^{2/3} = \sqrt[3]{(-2)^2} = \sqrt[3]{4}$

$f'(x) = \frac{2}{\cancel{3}}(x^3-1)^{-1/3} \cdot \cancel{3}x^2$

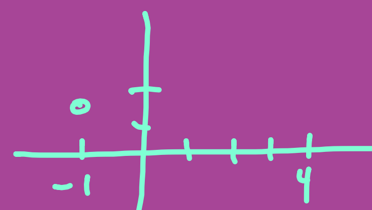
$$\frac{2x^2}{\sqrt[3]{x^3-1}} = 0$$

$2x^2 = 0$

$x = 0$

pt of nondifferentiability $\longrightarrow$ $x = 1$

must check pts are in interval.



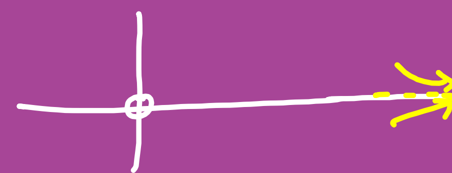
| | |
|---|---|
| 0 | $-1^{2/3} = \sqrt[3]{(-1)^2} = 1$ |
| 1 | 0 |
| 4 | $63^{2/3} = \sqrt[3]{3969} \approx 15.83$ |

$\lim_{x \to -1^+} f(x) = \sqrt[3]{4} \approx 1.\sim$

Abs max $(4, 63^{2/3})$

Abs min $(1, 0)$

$$f(x) = \frac{x}{x^2+1} \quad (0, \infty)$$



$$\lim_{x \to 0^+} \frac{x}{x^2+1} = \frac{0}{1} = 0$$

$$\lim_{x \to \infty} \frac{x}{x^2} = \lim_{x \to \infty} \frac{1}{x} = \frac{1}{\infty} = 0$$

OR

L'Hop. $\lim_{x \to \infty} \frac{x}{x^2+1} = \frac{\infty}{\infty}$

$$\lim_{x \to \infty} \frac{1}{2x} = \frac{1}{\infty} = 0$$

$$f'(x) = \frac{(x^2+1)\cdot 1 - x \cdot 2x}{(x^2+1)^2}$$

$$= \frac{x^2+1-2x^2}{(x^2+1)^2}$$

$$0 = \frac{1-x^2}{(x^2+1)^2}$$

$$0 = 1 - x^2$$

$$\sqrt{x^2} = \sqrt{1}$$

$$x = \cancel{\pm} 1$$

(-1 is not in interval)

| | |
|---|---|
| 1 | $\frac{1}{2}$ |

Abs max $(1, \frac{1}{2})$

No Abs min.

10/ $f(x) = \dfrac{x}{e^{2x}}$ $[1, \infty)$

$\lim_{x \to \infty} \dfrac{x}{e^{2x}} = \dfrac{\infty}{\infty}$

$\lim_{x \to \infty} \dfrac{1}{e^{2x} \cdot 2} = \dfrac{1}{\infty} = 0$

| | |
|---|---|
| 1 | $\frac{1}{e^2} \approx 0.$ —— |

Abs max $(1, 1/e^2)$
No Abs min.

$f'(x) = \dfrac{e^{2x} \cdot 1 - x \cdot e^{2x} \cdot 2}{(e^{2x})^2}$

$= \dfrac{e^{2x}[1-2x]}{(e^{2x})^2}$

$0 = \dfrac{1-2x}{e^{2x}}$

$0 = 1 - 2x$

$2x = 1$

$x = \cancel{1/2}$ ← not in interval