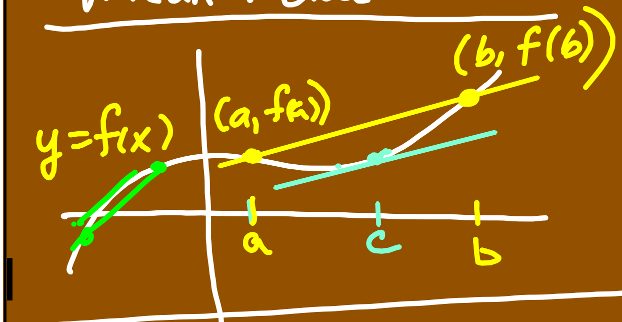
CURVE SKETCHING

Mean Value Theorem



$m = \frac{f(b)-f(a)}{b-a}$

1) f is continuous on [a,b]
2) f is differentiable on (a,b)
3) $f'(c) = \frac{f(b)-f(a)}{b-a}$

$f(x) = x^3 - 3x^2 + 2x \quad [\underset{a}{0}, \underset{b}{2}]$

Find point c.

$f'(x) = 3x^2 - 6x + 2$

$3c^2 - 6c + 2 = \frac{f(2)-f(0)}{2-0}$

$f(2) = 8-12+4 = 0$

$f(0) = 0-0+0 = 0$

$3c^2 - 6c + 2 = \frac{0-0}{2}$

$3c^2 - 6c + 2 = 0$

$c = \frac{6 \pm \sqrt{36-4(3)(2)}}{2(3)}$

$= \frac{6 \pm \sqrt{12}}{6} = \frac{6 \pm 2\sqrt{3}}{6}$

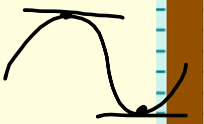
$= \frac{3 \pm \sqrt{3}}{3} \approx 1.6 \quad \approx 0.4$

$c = \frac{3+\sqrt{3}}{3}$ OR $\frac{3-\sqrt{3}}{3}$

must be in [0,2]

CURVE SKETCHING

$f(x) = x^4 - 4x^3 + 10$

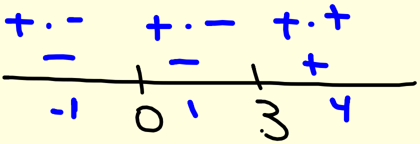


1) Find inc & dec

$f'(x) = 4x^3 - 12x^2 = 0$

$\Rightarrow = 4x^2(x-3) = 0$

$x = 0, 3 \leftarrow$ critical pts.



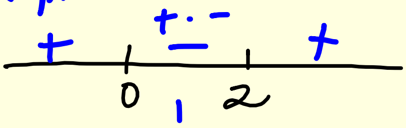
2) Concavity

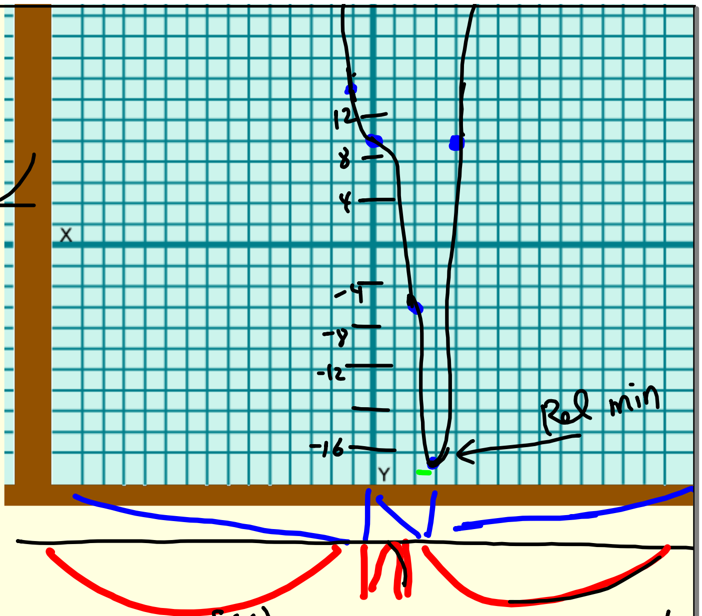
$f''(x) = 12x^2 - 24x = 0$

$= 12x(x-2)$

$x = 0, 2$

possible infl pts.

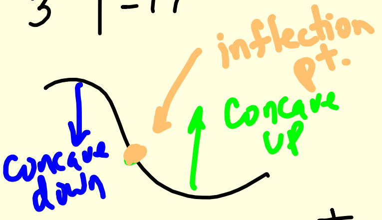




	f(x)
0	10
2	-6
3	-17

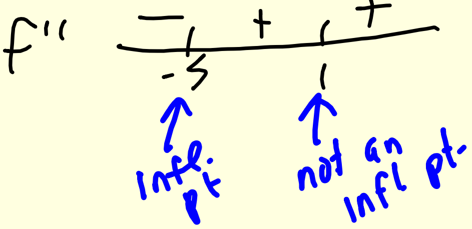
-1	15
4	10

Infl. pt.
(0,10)
(2,-6)



crit pt.

$f'(x) = 0$



$f(x) = 3x^{2/3} - x$

1) $f'(x) = 2x^{-1/3} - 1 = 0$

$\cancel{\sqrt[3]{x}}\ \frac{2}{\cancel{\sqrt[3]{x}}} = 1 \cdot \sqrt[3]{x}$

$(2)^3 = (\sqrt[3]{x})^3$

$8 = x$

$f'(x) = \frac{2}{\sqrt[3]{x}} - 1 \leftarrow$ $2x^{-1/3} - 1$

$x \neq 0$

$-$ $+$ $-$

0 8

-1 0 1 8 27

2) $f''(x) = -\frac{2}{3}x^{-4/3} = 0$

$x = 0$ $\frac{-2}{3\sqrt[3]{x^4}} = 0$

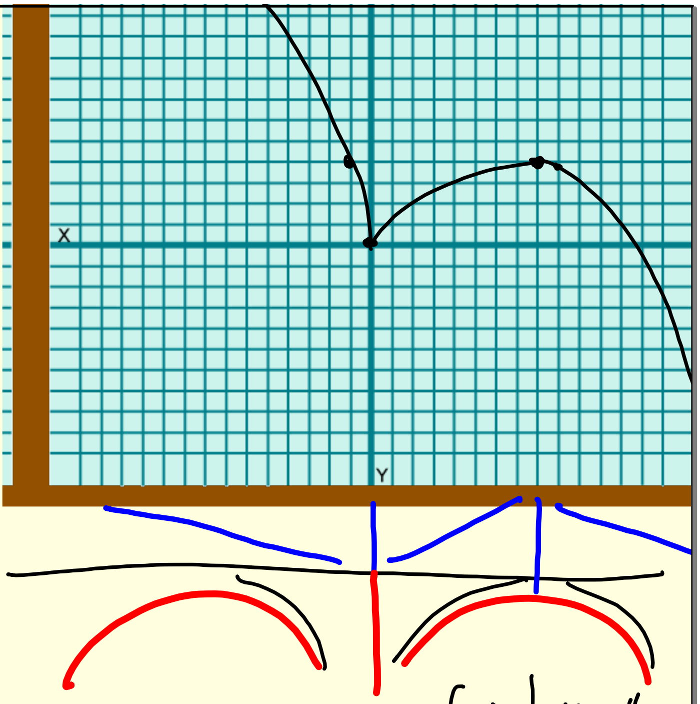
$-2 = 0$

$\frac{-}{+}$ $\frac{-}{+}$

$-$ $-$

0

-1 0 1



0	0
8	4
-1	4
9	3.98

$f(x)$ | $x = \#$

To find critical pts.

1) $f'(x)=0$

2) points of non-differentiability (special critical pts)

- where $f'(a)$ is undefined. $\left(\frac{\#}{0}\right)$
  $f(a)$ is defined.

November 9, 2022