Polynomial Graphs

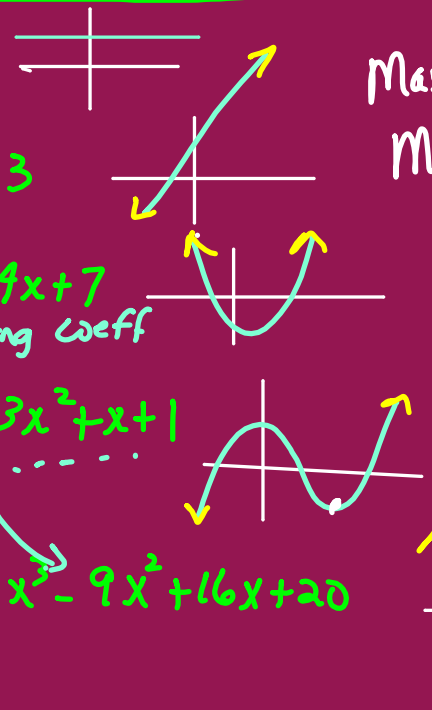
$f(x) = 2x^0$ or $y = 2$

$f(x) = 2x^1 + 3$ Linear

$f(x) = 3x^2 - 4x + 7$ Quadratic ↑ leading coeff

$f(x) = x^3 - 3x^2 + x + 1$ Cubic $-7x^3 \ldots$

$f(x) = x^4 - 4x^3 - 9x^2 + 16x + 20$ quartic



Degree = highest power

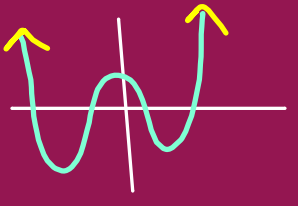
Max # of x-int = Degree

Max # of peaks/valley = Degree − 1

End Behavior

odd degree = ends go in opposite directions

even degree = ends go in same direction



Polynomial = many terms (one or more terms)

* Terms must be added or subtracted
* Exponents must be whole #'s 0, 1, 2, 3, 4 ...
* Coefficients are real #'s

P $5x^8 - 3x^5 + 7x^4 - 2x + 9$ ← Degree = 8

P $3x^2y^1 + 9x^6y^3 + 7x - 5$

No $\frac{2}{5x^2} + 3x - 1 = \frac{2}{5}x^{-2} + 3x - 1$

P $\frac{4}{3}x^7 - \sqrt{2}x^8 + \frac{3}{5}x - 4$ ← 8th degree

No $(4x^7 - 3)(5x^2 + 9)$ — No mult.

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Name_____

POLYNOMIALS HANDOUT

For each function, determine if it is a polynomial and then state the degree, the name, and the leading coefficient.

1. $f(x) = -3x + 5x^3 - 6x + 2$

2. $f(x) = 9x^4 + 8x^3 - 6x^{-2} - 1$

3. $f(x) = 3x^4 = 2x - \frac{5}{x} + 9x^2 - 7$

4. $f(x) = \frac{5}{3}x^2 - \sqrt{7}x^4 + 8x^3 - \frac{1}{2} + x$

Match each function and graph.

E 5. $f(x) = -3x^2 + 8x - 1$

A 6. $f(x) = -x^4 + x^3 + 4x^2 + 2x - 1$

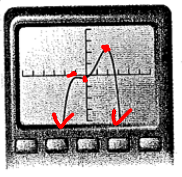
B 7. $f(x) = -2x^3 - 3x^2 + 7$

F 8. $f(x) = 4x - 5$

C 9. $f(x) = 2x^4 - 2x^3 - 5x^2 + 7x - 2$

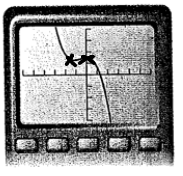
D 10. $f(x) = x^3 - 4x^2 - 3x + 2$

A.



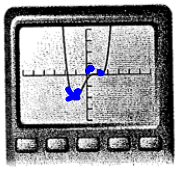
even
$-x^{\text{even}}$
Deg ≥ 4

B.



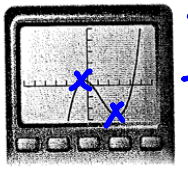
odd
−
Deg ≥ 3

C.



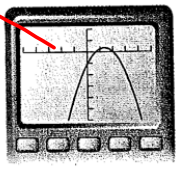
Deg ≥ 4

D.

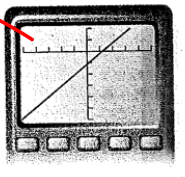


odd
Deg ≥ 3
+

E.



F.



11. $f(x) = 9x^3 - 4 + x^2$

12. $f(x) = 0.4x^2 - x + 3$

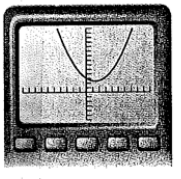
13. $f(x) = x^4 - 4x^3 + x^2 - 6$

14. $f(x) = -3x^3 - 8x^2 - x + 1$

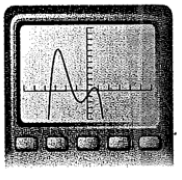
15. $f(x) = 8 - x$

16. $f(x) = -x^4 - 4x^3 + x^2 + 6x - 2$

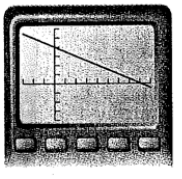
A.



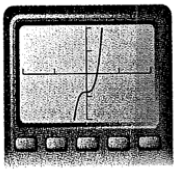
B.



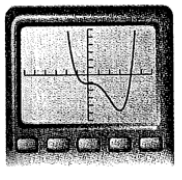
C.



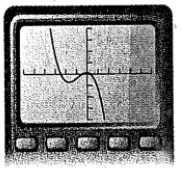
D.



E.

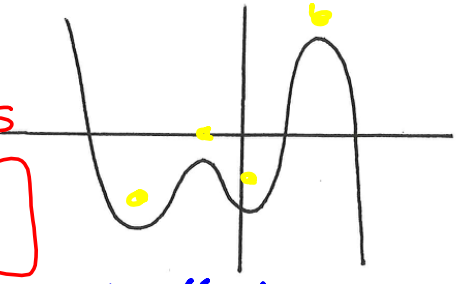


F.

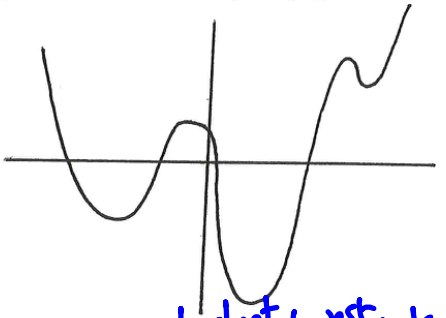


Describe the degree and sign of the leading coefficient of the polynomial function using the graph.

17. odd
Deg ≥ 5
$-x^5$



18.

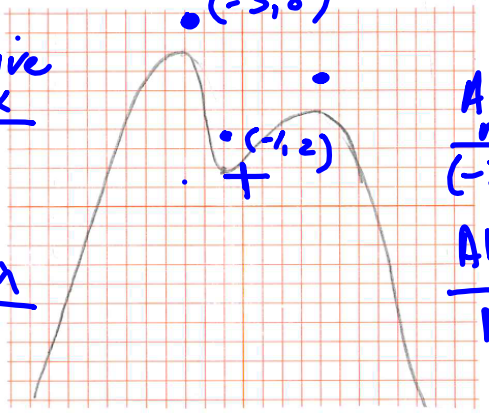


all peaks & valleys

highest & lowest points

For each function, identify (a) intervals where the function is increasing or decreasing, (b) coordinates of all relative maximums and minimums, and (c) coordinates of all absolute maximums and minimums.

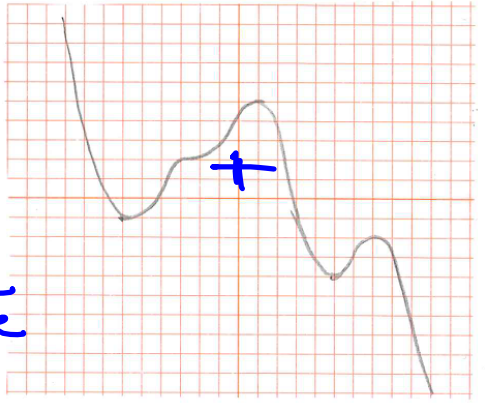
19.



Relative max
(-3,8)
(4,5)

Rel min
(-1,2)

20.



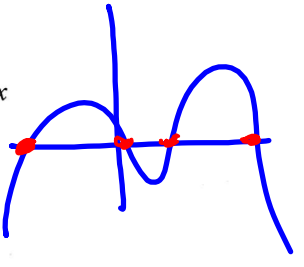
Abs max
(-3,8)

Abs min
None

Find the real zeros of each polynomial using your calculator. Round to hundredths.

21. $f(x)=x^4+2x^3-x-1$

22. $f(x)=-x^5+9x^3-9x$



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Polynomial Operations

$(4x^3 + 2x^7 + 3x - 9) + (-x^6 + -7x^7 + 4x^3 + -8)$

$-5x^7 - x^6 + 8x^3 + 3x - 17$

$(2x+3)(4x-1)(x+5)$

$(8x^2 - 2x + 12x - 3)$

$(8x^2 + 10x - 3)(x+5)$

$8x^3 + 10x^2 - 3x + 40x^2 + 50x - 15$

$= 8x^3 + 50x^2 + 47x - 15$

Special Cases

$(4x+3)(4x-3)$ ← Do Firsts + Lasts

$16x^2 - 12x + 12x - 9$

$= 16x^2 - 9$

$(3x-7)^2 = 9x^2 - 49$

$(3x-7)(3x-7)$

$9x^2 - 21x - 21x + 49$

$9x^2 - 42x + 49$

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FACTORING

★ First Step = Pull out common factors

2 terms = look for perfect squares or perfect cubes

$a^2 - b^2 = (a+b)(a-b)$ ← conjugates

$a^2 + b^2 = a^2 + b^2$ ← unfactorable

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

square, multiply, square

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$x^3 - 64 = (x-4)(x^2 + 4x + 16)$

$64 = 4^3$

$1^3 = 1$
$2^3 = 8$
$3^3 = 27$
$4^3 = 64$
$5^3 = 125$
$6^3 = 216$
$7^3 = 343$

3 terms – UNFOIL

$x^2+2x-15$

$(x+5)(x-3)$

4 terms – Grouping

$(3x^3+15x^2-2x-10)$

1) $3x^2(x+5)-2(x+5)$

must be the same

2) $(x+5)(3x^2-2)$

2 steps

1) Group each pair of terms
   ↓ pull out the common factor
2) Pull out the Common quantity

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