Semester Review

Limits

- $x \to \#$: Sub # in
  - $\frac{0}{0}$: factor, conjugate, L'Hopital's
  - $\frac{\#}{0}$: test pts for $\pm\infty$
- $x \to \pm\infty$
  - polyn: Use highest power term
  - Rational: highest power from num & denom
- Other: e^x, $\ln x$, Check

$$\lim_{x \to +\infty} e^x = +\infty \qquad \lim_{x \to -\infty} e^x = 0$$

$$\lim_{x \to +\infty} \ln x = +\infty \qquad \lim_{x \to 0^+} \ln x = -\infty$$

$\frac{0}{0}$, $\frac{\infty}{\infty}$, $\infty - \infty$ make fraction, Do L'Hop

0^0, 1^∞, ∞^0

$e^{\ln x^x}$

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

$$\lim_{x \to 0} \frac{1 - \cos x}{x} = 0$$

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Limits

$$\lim_{x\to -\infty} \frac{\sqrt{9x^2-2x}}{6x+1}$$

$$\lim_{x\to -\infty} \frac{\sqrt[2]{9x^2}}{6x}$$ even

$$\lim_{x\to -\infty} \frac{3|x|}{6x}$$ odd

$$\lim_{x\to -\infty} \frac{-3\cancel{x}}{6\cancel{x}} = -\frac{1}{2}$$

$$\lim_{x\to 0^+} x^{x^2} = 0^0$$

$$\lim_{x\to 0^+} e^{\ln x^{x^2}}$$

$$\lim_{x\to 0^+} x^2 \cdot \ln x = 0\cdot -\infty$$

$$\lim_{x\to 0^+} \frac{\ln x}{x^{-2}} = \frac{-\infty}{\infty}$$

$$\lim_{x\to 0^+} \frac{\frac{1}{\cancel{x}}}{\frac{-2}{\cancel{x^3}}} \cdot \frac{x^{\cancel{3}\,2}}{-2}$$

$$\lim_{x\to 0^+} \frac{x^2}{-2} = \frac{0}{-2} = 0$$

$$e^0 = \boxed{1}$$

Continuity (13) / Differentiability (17)

$$f(x) = \begin{cases} \dfrac{x^3-8}{x-2} & x < 2 \\ 4+4x & x \ge 2 \end{cases}$$

$a = 2$

1) $f(c)$ is defined
2) $\lim_{x \to a} f(x)$ exists
3) $f(c) = \lim_{x \to 2} f(x)$

Differentiability
4) $f'(a)^- = f'(c)^+$

1) $f(2) = 4 + 4(2) = 12$

2) * $\lim_{x \to 2^-} \dfrac{x^3-8}{x-2} = \dfrac{0}{0}$

$\lim_{x \to 2^-} \dfrac{3x^2}{1} = 12$

* $\lim_{x \to 2^+} 4 + 4x = 12$

* $\lim_{x \to 2} f(x) = 12$

3) $f(2) = \lim_{x \to 2} f(x)$

Continuous

4) $f'(2)^- = \dfrac{(x-2)\cdot 3x^2 - (x^3-8)\cdot 1}{(x-2)^2}$

$= \dfrac{\cancel{(x-2)}\cdot 3x^2 - \cancel{(x-2)}(x^2+2x+4)}{(x-2)^{\cancel{2}}}$

$= \dfrac{12 - (12)}{x-2} = \dfrac{0}{0}$

$f'(2)^+ = 4$

$f'(2)^+ \neq f'(2)^-$

not differentiable

$$\frac{d}{dx}\sin x = \cos x \qquad \frac{d}{dx}\cos x = -\sin x$$

$$\frac{d}{dx}\tan x = \sec^2 x \qquad \frac{d}{dx}\cot x = -\csc^2 x$$

$$\frac{d}{dx}\sec x = \sec x \tan x \qquad \frac{d}{dx}\csc x = -\csc x \cot x$$

$$\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\tan^{-1} x = \frac{1}{x^2+1} \qquad \frac{d}{dx}\sec^{-1} x = \frac{1}{|x|\sqrt{x^2-1}}$$

$$\frac{d}{dx}e^x = e^x \qquad \frac{d}{dx}\ln x = \frac{1}{x} \qquad \frac{d}{dx}a^x = \ln a \cdot a^x$$

$$f(x) = 6^{x^3-3x^2}$$

$$f'(x) = \ln 6 \cdot 6^{x^3-3x^2} \cdot (3x^2-6x)$$

Definitions of Derivative

$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$f(x) = x^2 - 2x + 3$$

$$\lim_{x \to a} \frac{x^2 - 2x + \cancel{3} - (a^2 - 2a + \cancel{3})}{x - a}$$

$$\lim_{x \to a} \frac{x^2 - a^2 - 2x + 2a}{x - a}$$

$$\lim_{x \to a} \frac{(x+a)\cancel{(x-a)} - 2\cancel{(x-a)}}{\cancel{x-a}}$$

$$= a + a - 2$$

$$= 2a - 2$$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h \to 0} \frac{(x+h)^2 - 2(x+h) + 3 - (x^2 - 2x + 3)}{h}$$

$$(x+h)(x+h)$$

$$\lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{2x} - 2h + \cancel{3} - \cancel{x^2} + \cancel{2x} - \cancel{3}}{h}$$

$$\lim_{h \to 0} \frac{2xh + h^2 - 2h}{h}$$

$$\lim_{h \to 0} \frac{\cancel{h}(2x + h - 2)}{\cancel{h}}$$

$$= 2x + 0 - 2$$

$$= 2x - 2$$