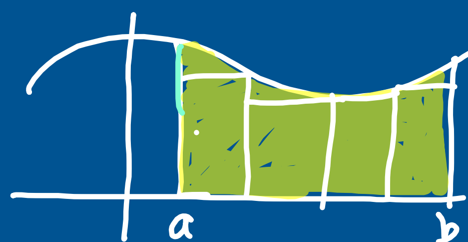
Riemann Sums



$f(x) = 3x^2 + 4$ $[1, 3]$

4 sub-intervals

$\text{width} = \frac{b-a}{n}$

$\text{Width} = \frac{3-1}{4}$

$= 1/2$

Right =

$\frac{1}{2}[f(1.5)+f(2)+f(2.5)+f(3)]$

$\frac{1}{2}[10.75+16+22.75+31]$

$= \frac{1}{2}[80.5]$

$= 40.25 \text{ units}^2$

Left $= f(1)\cdot\frac{1}{2} + f(1.5)\cdot\frac{1}{2} + f(2)\cdot\frac{1}{2} + f(2.5)\frac{1}{2}$

$= \frac{1}{2}[f(1)+f(1.5)+f(2)+f(2.5)]$

$= \frac{1}{2}[7 + 10.75 + 16 + 22.75]$

$= \frac{1}{2}[56.5]$

$= 28.25 \text{ units}^2$

$$\int_{-2}^{5} (4x+3)\,dx$$

$$= \frac{4x^2}{2} + 3x + C \Big|_{-2}^{5}$$

$$= 2x^2 + 3x + C \Big|_{-2}^{5}$$

$$= 2(5)^2 + 3(5) + C - \left(2(-2)^2 + 3(-2) + C\right)$$

$$= 50 + 15 + \cancel{C} + (-8 + 6 \cancel{- C})$$

$$= 63$$

$$\int_{1}^{6} x\sqrt{x+3}\,dx$$

$u = x+3$ $\quad u-3 = x$

$du = dx$

Change limits of integration

$u = x+3$

$u = 6+3 = 9$

$u = 1+3 = 4$

$$\int x \cdot u^{1/2} \cdot du$$

$$\int_{4}^{9} (u-3) \cdot u^{1/2}\,du$$

$$\int_{4}^{9} (u^{3/2} - 3u^{1/2})\,du$$

$$\frac{2}{5}u^{5/2} - \frac{2\cancel{3}}{\cancel{3}}u^{3/2} \Big|_{4}^{9}$$

$$\frac{2}{5}u^{5/2} - 2u^{3/2} \Big|_{4}^{9}$$

$$\frac{2}{5}(9)^{5/2} - 2(9)^{3/2} + \left[-\frac{2}{5}(4)^{5/2} + 2(4)^{3/2}\right]$$

$\sqrt[2]{9^5}$ $\quad 3^5 = 243$ $\quad \sqrt{9^3}$ $\quad 3^3$ $\quad \cdot \frac{2}{5}\sqrt{4^5}$ $\quad \frac{2}{5} \cdot 32$ $\quad 2\sqrt{4^3}$ $\quad 2 \cdot 8$

When using u-sub, change the limits of integ to u-values!

$$= \frac{486}{5} - 54 - \frac{64}{5} + 16$$

$$= \frac{422}{5} - 38$$

$$= \frac{422}{5} - \frac{190}{5} = \frac{232}{5}$$

FUNDAMENTAL THEOREM OF CALCULUS

Part 1

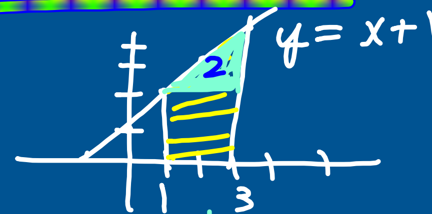
$$\int_1^3 (x+1)\,dx$$

$$\left.\frac{x^2}{2} + x\,\right|_1^3$$

$$\frac{9}{2} + 3 - \left[1/2 + 1\right]$$

$$= 4 + 2$$

$$= 6$$

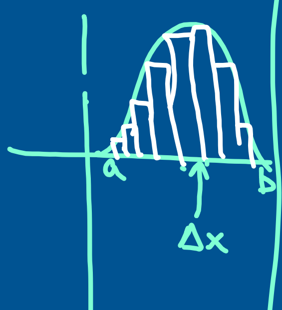


$$\Delta = \frac{1}{2}bh = \frac{1}{2}\cdot 2\cdot 2 = 2$$

$$\square = b\cdot h = 2\cdot 2 = 4$$

$$\boxed{6}$$

Integration represents the area between a curve & an axis.

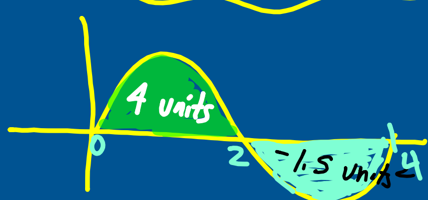


$$\boxed{\lim_{\Delta x \to 0} \sum_{x=a}^{b} f(x)\,\Delta x = \int_a^b f(x)\,dx}$$

$l \cdot w$

let width of rectangle go to 0.

sum area of rectangles



$$\int_0^2 f(x)\,dx = 4$$

$$\int_0^4 f(x)\,dx = 4 + -1.5 = 2.5$$

$$3\int_2^4 f(x)\,dx = 3(-1.5) = -4.5$$

$$\int_4^2 f(x)\,dx = -\int_2^4 f(x)\,dx$$

$$= -(-1.5) = 1.5$$

Part 2

$$\frac{d}{dx}\int_1^x (4t^2+t)\,dt = 4x^2+x$$

$$\frac{d}{dx}\int_6^x \frac{\sin^8(3t^2-1)}{\ln 8t^4}\,dt = \frac{\sin^8(3x^2-1)}{\ln 8x^4}$$

$$\frac{d}{dx}\int_2^{x^2} \frac{4}{\sqrt{t^3+2}}\,dt = \frac{4}{\sqrt{(x^2)^3+2}}\cdot 2x$$

$$= \frac{4}{\sqrt{x^6+2}}\cdot 2x$$

$$\frac{d}{dx}\int_{x^4}^{3x^7} \frac{2t}{t+1}\,dt$$

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