ANTIDIFFERENTIATION

= Integration (Area under a curve)

$y = f(x)$

$\frac{dy}{dx} = f'(x)$

$\int dy = \int f'(x)\, dx$

integral sign

$\int (x^2 - 2x)\, dx$

$\int (4x^1 + 9x^2)\, dx$

$\frac{4x^2}{2} + \frac{9x^3}{3}$

$\boxed{2x^2 + 3x^3 + C}$

Power Rule

$\int x^n\, dx = \frac{x^{n+1}}{n+1} + C$

$$\int \left(\frac{2}{x^3} + 4\sqrt[3]{x} - x^{3/5} + 7\right) dx$$

- all vars in numerator
- Individual terms with + or −

$$= \int 2x^{-3} + 4x^{1/3} - x^{3/5} + 7)\,dx$$

$$= \frac{\cancel{2}x^{-2}}{-\cancel{2}} + \frac{3}{\cancel{4}}\cancel{4}x^{4/3} - \frac{5}{8}x^{8/5} + 7x + C$$

$$= \frac{-1}{x^2} + 3x^{4/3} - \frac{5}{8}x^{8/5} + 7x + C$$

$$\int (7x^2-4)^2\,dx$$

$$\int (7x^2-4)(7x^2-4)\,dx$$

$$\int (49x^4 - 56x^2 + 16)\,dx$$

$$= \frac{49x^5}{5} - \frac{56x^3}{3} + 16x + C$$

$$\int \frac{4x^2-2x+1}{\sqrt{x}}\,dx$$

$$\int (4x^2-2x^1+1)\,x^{-1/2}\,dx$$

$$\int \left(4x^{3/2}-2x^{1/2}+x^{-1/2}\right)dx$$

$$= \frac{2}{5}\cdot 4x^{5/2} - \frac{2}{3}2x^{3/2} + 2x^{1/2} + C$$

$$= \boxed{\frac{8}{5}x^{5/2} - \frac{4}{3}x^{3/2} + 2x^{1/2} + C}$$

Initial value problem.

Find y.

$$\int \frac{dy}{dx} = \int (3x^2+2x)\,dx \qquad y(2)=7$$

$x \quad y$

$$y = \cancel{\frac{3x^3}{3}} + \cancel{\frac{2x^2}{2}} + C$$

$$y = x^3 + x^2 + C$$

$$7 = 2^3 + 2^2 + C$$

$$7 = 8 + 4 + C$$

$$-12$$

$$-5 = C$$

$$\boxed{y = x^3 + x^2 - 5}$$

U-Substitution

$$\int 6x(x^2+5)^8\,dx$$

$$u = x^2+5$$

$$dx\,\frac{du}{\cancel{dx}} = 2x\,dx$$

$$\frac{du}{2x} = dx$$

$$\int 6x \cdot u^8 \cdot dx$$

$$\int \overset{3}{\cancel{6x}} \cdot u^8 \cdot \frac{du}{\cancel{2x}}$$

$$\int 3u^8\,du$$

$$= \frac{3u^9}{9} + C$$

$$= \boxed{\frac{1}{3}(x^2+5)^9 + C}$$

$$\int \frac{3x}{(4-3x^2)^7}\,dx$$

$$u = 4-3x^2$$

$$du = -6x\,dx$$

$$\frac{du}{-6x} = dx$$

$$\int \frac{\cancel{3x}}{u^7}\,\frac{du}{\underset{-2}{\cancel{-6x}}}$$

$$-\frac{1}{2}\int u^{-7}\,du$$

$$+\frac{1}{2}\,\frac{u^{-6}}{+6} + C$$

$$\frac{1}{12u^6} + C$$

$$\frac{1}{12(4-3x^2)^6} + C$$

$$\frac{d}{dx}\sin x = \cos x$$
$$\frac{d}{dx}\cos x = -\sin x$$
$$\frac{d}{dx}\tan x = \sec^2 x$$
$$\frac{d}{dx}\cot x = -\csc^2 x$$
$$\frac{d}{dx}\sec x = \sec x \tan x$$
$$\frac{d}{dx}\csc x = -\csc x \cot x$$
$$\frac{d}{dx}e^x = e^x$$
$$\frac{d}{dx}\ln x = \frac{1}{x}$$

$$\int \cos x\,dx = \sin x + C$$
$$\int \sin x\,dx = -\cos x + C$$
$$\int \sec^2 x\,dx = \tan x + C$$
$$\int \csc^2 x\,dx = -\cot x + C$$
$$\int \sec x \tan x\,dx = \sec x + C$$
$$\int \csc x \cot x\,dx = -\csc x + C$$
$$\int e^x\,dx = e^x + C$$
$$\int \frac{1}{x}\,dx = \ln|x| + C$$

$$\int (4\cos x - 3\sec^2 x + 4\csc x\cot x)\,dx$$
$$= 4\sin x - 3\tan x - 4\csc x + C$$

$$\int \csc x(\csc x - \cot x)\,dx$$
$$\int (\csc^2 x - \csc x\cot x)\,dx$$
$$-\cot x + \csc x + C$$

$$\int \left(\frac{1}{\sin^2 x} - \cot x \sin x\right) dx$$
$$= \int \left(\csc^2 x - \frac{\cos x}{\cancel{\sin x}} \cancel{\sin x}\right) dx$$
$$= -\cot x - \sin x + C$$

$$\int \left(\frac{6}{x} + 5e^x\right) dx$$
$$\int 6 \cdot \frac{1}{x}$$
$$6\ln|x| + 5e^x + C$$