MORE INTEGRATION

$$\int 5x\sqrt{2x+3}\, dx$$

$$U = 2x+3 \quad\quad \frac{U-3}{2} = \frac{2x}{2}$$
$$du = 2\, dx$$
$$\frac{du}{2} = dx$$

$$\int 5x \cdot u^{1/2} \cdot \frac{du}{2}$$

$$\frac{5}{2}\int \frac{u-3}{2} \cdot u^{1/2}\, du$$

$$\frac{5}{4}\int \left(u^{3/2} - 3u^{1/2}\right) du$$

$$\frac{5}{4}\left[\frac{2}{5}u^{5/2} - \frac{3\cdot 2}{3}u^{3/2}\right] + C$$

$$\frac{1}{2}u^{5/2} - \frac{5}{2}u^{3/2} + C$$
$$\frac{1}{2}(2x+3)^{5/2} - \frac{5}{2}(2x+3)^{3/2} + C$$

$$\int (x+2)^2 \sqrt{1+x} \; dx \qquad\qquad u = 1+x \Rightarrow u-1 = x$$
$$\qquad\qquad\qquad\qquad\qquad\qquad du = 1 \cdot dx$$

$$\int (x+2)^2 \cdot u^{1/2} \cdot du$$

$$\int (u-1+2)^2 \, u^{1/2} \, du$$

$$\int (u+1)^2 \cdot u^{1/2} \, du$$

$$\int (u^2 + 2u + 1) \, u^{1/2} \, du$$

$$\int \left(u^{5/2} + 2u^{3/2} + u^{1/2} \right) du$$

$$= \frac{2}{7} u^{7/2} + \frac{2 \cdot 2}{5} u^{5/2} + \frac{2}{3} u^{3/2} + C$$

$$= \frac{2}{7} (1+x)^{7/2} + \frac{4}{5} (1+x)^{5/2} + \frac{2}{3} (1+x)^{3/2} + C$$

$$\int \tan^8 x \; \sec^2 x \; dx \qquad u = \tan x$$

$$\int u^8 \cdot \cancel{\sec^2 x} \; \frac{du}{\cancel{\sec^2 x}} \qquad du = \sec^2 x \; dx$$

$$\frac{du}{\sec^2 x} = dx$$

$$\int u^8 \, du$$

$$= \frac{u^9}{9} + C$$

$$= \frac{\tan^9 x}{9} + C$$

$$\int x^6 \sin(x^7)\, dx \qquad u = x^7$$

$$\int x^6 \sin u \cdot \frac{du}{7x^6} \qquad du = 7x^6\, dx$$

$$\frac{du}{7x^6} = dx$$

$$\frac{1}{7} \int \sin u \, du$$

$$-\frac{1}{7} \cos u + C$$

$$-\frac{1}{7} \cos x^7 + C$$

$$\int \frac{1}{y^2} \sec\left(\frac{1}{y}\right) \tan\left(\frac{1}{y}\right) dy$$

$$\int \frac{1}{y^2} \sec u \, \tan u \cdot -y^2 \, du$$

$$-\int \sec u \tan u \, du$$

$$= -\sec u + C$$

$$= -\sec\left(\frac{1}{y}\right) + C$$

$$U = \frac{1}{y} = y^{-1}$$

$$du = -1 y^{-2} dy$$

$$du = -\frac{1}{y^2} dy$$

$$-y^2 du = dy$$

$$\int \frac{8\cos(4x-7)}{\sin^6(4x-7)} \, dx \qquad u = \sin(4x-7)$$

$$du = \cos(4x-7)\cdot 4 \, dx$$

$$\int \frac{\overset{2}{\cancel{8\cos(4x-7)}}}{u^6} \cdot \frac{du}{\cancel{4\cos(4x-7)}} \qquad \frac{du}{4\cos(4x-7)} = dx$$

$$= \int 2u^{-6} \, du$$

$$= \frac{2u^{-5}}{-5} + C$$

$$= \frac{-2}{5u^5} + C$$

$$\boxed{\frac{-2}{5\sin^5(4x-7)} + C}$$