Integration with e^x, a^x, $\ln x$, & inv. trig functions

$\frac{d}{dx} e^x = e^x \quad \int e^x\,dx = e^x + C$

$\frac{d}{dx} \ln x = \frac{1}{x} \quad \int \frac{1}{x}\,dx = \ln|x| + C$

$\frac{d}{dx} a^x = \ln a \cdot a^x \quad \int a^x\,dx = \frac{1}{\ln a} \cdot a^x + C = \frac{a^x}{\ln a} + C$

$\int x \cdot e^{5x^2}\,dx$

$u = 5x^2$

$du = 10x\,dx$

$\frac{du}{10x} = dx$

$\int \cancel{x} \cdot e^u \cdot \frac{du}{10\cancel{x}}$

$\frac{1}{10} \int e^u\,du$

$\frac{1}{10} e^u + C$

$\boxed{\frac{1}{10} e^{5x^2} + C}$

$\int \frac{e^{\tan y}}{\cos^2 y}\,dy$

$u = \tan y$

$du = \sec^2 y\,dy$

$\frac{du}{\sec^2 y} = dy$

$\int \frac{e^u}{\cos^2 y} \cdot \frac{du}{\sec^2 y}$

$\int \frac{e^u}{\cancel{\cos^2 y}} \cdot \frac{du}{\frac{1}{\cancel{\cos^2 y}}}$

$\int e^u\,du$

$= e^u + C$

$= e^{\tan y} + C$

$$\int a^x\,dx = \frac{1}{\ln a}a^x + C$$

$$\int 13^x\,dx = \frac{1}{\ln 13}\cdot 13^x + C$$

$$\int 3x^5 \cdot 7^{x^6}\,dx$$

$$u = x^6$$
$$du = 6x^5\,dx$$
$$\frac{du}{6x^5} = dx$$

$$\int \cancel{3}\cancel{x^5}\cdot 7^u \frac{du}{2\,\cancel{6}\,\cancel{x^5}}$$

$$\frac{1}{2}\int 7^u\,du$$

$$= \frac{1}{2}\cdot\frac{1}{\ln 7}\cdot 7^u + C$$

$$= \frac{1}{2\ln 7}\cdot 7^{x^6} + C$$

$$\int \frac{x^2}{1-x^3}\,dx$$

$u = 1-x^3$

$du = -3x^2\,dx$

$\frac{du}{-3x^2} = dx$

$\int \frac{1}{x} = \ln|x| + C$

$$\int \frac{\cancel{x^2}}{u}\cdot\frac{du}{-3\cancel{x^2}}$$

$$-\frac{1}{3}\int \frac{1}{u}\,du$$

$$-\frac{1}{3}\ln|u| + c$$

$$-\frac{1}{3}\ln|1-x^3| + C$$

$\ln|x^4|$

$$\int \frac{\csc^2 x}{\cot x}\,dx$$

$u = \cot x$

$du = -\csc^2 x\,dx$

$\frac{du}{-\csc^2 x} = dx$

$$\int \frac{\cancel{\csc^2 x}}{u}\cdot\frac{du}{-\cancel{\csc^2 x}}$$

$$-\int \frac{1}{u}\,du$$

$$= -\ln|u| + C$$

$$= -\ln|\cot x| + C$$

$$\int \frac{(\ln z)^5}{z}\,dz$$

$u = \ln z$

$du = \frac{1}{z}\,dz$

$$\int \frac{u^5}{\cancel{z}}\,\cancel{z}\cdot du$$

$z \cdot du = dz$

$$= \frac{u^6}{6} + C$$

$$= \frac{(\ln z)^6}{6} + C$$

Inverse Trig Functions

$$\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}}$$

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}x + C$$

$$\frac{d}{dx}\tan^{-1}x = \frac{1}{x^2+1}$$

$$\int \frac{1}{x^2+1}\,dx = \tan^{-1}x + C$$

$$\frac{d}{dx}\sec^{-1}x = \frac{1}{|x|\sqrt{x^2-1}}$$

$$\int \frac{1}{x\sqrt{x^2-1}}\,dx = \sec^{-1}x + C$$

$$\int \frac{6x^2}{\sqrt{1-25x^6}}\,dx$$

$u = 5x^3$

$du = 15x^2\,dx$

$\frac{du}{15x^2} = dx$

$(5x^3)^2$

$$\int \frac{6\cancel{x^2}}{\sqrt{1-u^2}} \cdot \frac{du}{15\cancel{x^2}}$$

$$\frac{6}{15}\int \frac{1}{\sqrt{1-u^2}}\,du$$

$\frac{2}{5}\sin^{-1}u + C$

$\frac{2}{5}\sin^{-1}(5x^3) + C$

1) Make the "1" by pulling out a #

2) u-sub for quantity to be squared

3) Cancel & integrate with inv. trig func.

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$$\int \frac{3x}{4+9x^4}\,dx$$

$$\frac{3}{4}\int \frac{x}{1+\frac{9}{4}x^4}\,dx \qquad \left(\tfrac{3}{2}x^2\right)^2$$

$u = \frac{3}{2}x^2$

$du = 3x\,dx$

$\frac{du}{3x} = dx$

$$\frac{\cancel{3}}{4}\int \frac{\cancel{x}}{1+u^2}\,\frac{du}{\cancel{3x}}$$

$$\frac{1}{4}\int \frac{1}{1+u^2}\,du$$

$$\frac{1}{4}\tan^{-1}u + C$$

$$\frac{1}{4}\tan^{-1}\left(\frac{3}{2}x^2\right) + C$$

$$\int \frac{4\cos x}{\sin x\sqrt{\sin^2 x-36}}\,dx$$

$$\frac{4}{6}\int \frac{\cos x}{\sin x\sqrt{\frac{\sin^2 x}{36}-1}}\,dx$$

$$u=\frac{\sin x}{6} \Rightarrow 6u=\sin x$$

$$u=\frac{1}{6}\sin x$$

$$du=\frac{1}{6}\cos x\,dx$$

$$\frac{6\,du}{\cos x}=dx$$

$$\frac{2}{3}\int \frac{\cancel{\cos x}}{\cancel{6}u\sqrt{u^2-1}}\cdot\frac{\cancel{6}\,du}{\cancel{\cos x}}$$

$$\frac{2}{3}\int \frac{1}{u\sqrt{u^2-1}}\,du$$

$$\frac{2}{3}\sec^{-1}u + C$$

$$\frac{2}{3}\sec^{-1}\left(\frac{1}{6}\sin x\right)+C$$