EXPONENTS & ROOTS

INVERSE FUNCTIONS

Function – Each x-coordinate is paired with Exactly one y-coord.

vertical line test

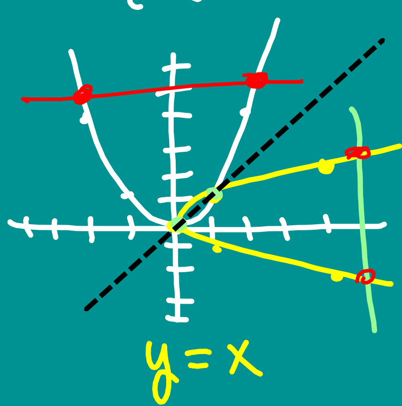
Inverse functions

$f = \{(x, y)\}$

$f^{-1} = \{(y, x)\}$

$f = \{(3,2)\ (-7,5)\ (4,-11)\}$

$f^{-1} = \{(2,3)\ (5,-7)\ (-11,4)\}$



$y = x$

$y = x^2$

0	0
±1	1
±2	4
±3	9

0	0
1	±1
4	±2
9	±3

Original function must pass the horiz line test for the f^{-1} to pass the vertical line test.

$y = x^3$

Find eq. of inverse.

1) Switch x + y variables.

2) Solve for y.

$f(x) = 4x - 7$

$y = 4x - 7$

$x = 4y - 7$

$\frac{x+7}{4} = \frac{4y}{\cancel{4}}$

$\frac{x+7}{4} = f^{-1}$

$\cancel{f(x)} = \sqrt[3]{2x+7}$ y

$(x)^3 = \left(\sqrt[3]{2y+7}\right)^3$

$x^3 = 2y + 7$

$\frac{x^3-7}{2} = \frac{\cancel{2}y}{\cancel{2}}$

$\frac{x^3-7}{2} = f^{-1}$

$f = \sqrt[3]{2x+7}$

$g = \frac{x^3-7}{2}$

$f \circ g = \sqrt[3]{\cancel{2}\left(\frac{x^3-7}{\cancel{2}}\right)+7}$

$\sqrt[3]{x^3 \cancel{-7+7}}$

$= x$

Given $f(x) = \sqrt{2x-5}$ $g(x) = \frac{x^2+5}{2}$

Are f + g inverse functions?

If $f \circ g$ or $g \circ f = x$, then f + g are inverses.

$g \circ f = \frac{\left(\sqrt{2x-5}\right)^2 + 5}{2}$

$= \frac{\cancel{2}x \cancel{-5} \cancel{+5}}{\cancel{2}}$

$= x$

yes, f + g are inverses

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RULES OF EXPONENTS

$3x^7$ ← exponent

x ← base

3 ← coefficient

7^5 ← base

Rule #1: $a^m \cdot a^n = a^{m+n}$

$x^3 \cdot x^7 = x^{10}$

$(a^3 b^5 c^4)(a^3 b^7 c^9)$

$= a^6 b^{12} c^{13}$

$7^2 \cdot 7^9 = 7^{11}$

$(2^5 \cdot 3^2)(2^2 \cdot 3^3)$

$= \boxed{2^7 \cdot 3^5}$

$128 \cdot 243 = 31{,}104$

NEVER CHANGE THE BASE!

RULE #2 = $(a^m)^n = a^{m \cdot n}$

$(K^3)^5 = K^{15}$

$K^3 \cdot K^3 \cdot K^3 \cdot K^3 \cdot K^3$

$(2^1 p^3 q^5)^6 = 2^6 p^{18} q^{30}$

$= 64 p^{18} q^{30}$

$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

$5^{-2} = \frac{1}{5^2} = \frac{1}{25}$

$\frac{1}{7^{-3}} = 7^3 = 343$

Rule #3: $\frac{a^m}{a^n} = a^{m-n}$

$\frac{x^6}{x^4} = x^{6-4} = x^2$

$\frac{f^2}{f^7} = f^{-5} = \frac{1}{f^5}$

Rule #4: $a^{-m} = \frac{1}{a^m}$

$\frac{1}{a^{-m}} = a^m$

$\frac{x^{-7}}{y^{-3}} = \frac{y^3}{x^7}$

RULE #5: $\boxed{a^0 = 1}$

$$\frac{f^7}{f^7} = f^0 = 1$$

$$2(xy^3)^0 + 5^0$$
$$= 2(1) + 1$$
$$= 2 + 1 = 3$$

$$\left(\frac{x^3}{y^2}\right)^{-4}$$
$$= \left(\frac{y^2}{x^3}\right)^4$$
$$= \frac{y^8}{x^{12}}$$

Shortcut

Flip fraction & change to positive exponent.

$$\frac{(2a^{7}b^{3}c^{-2})^{3}\cdot(2a^{-4}b^{-1}c^{5})^{-2}}{(2a^{-7}b^{11}c^{-5})^{2}\,\underbrace{\cancel{(2a^{41}b^{33}c^{105})^{0}}}_{1}}$$

$$=\frac{(2^{3}a^{21}b^{9}c^{-6})(2^{-2}a^{8}b^{2}c^{-10})}{(2^{2}a^{-14}b^{22}c^{-10}}$$

$$=\frac{\cancel{2^{1}}\,a^{29+14}\,\cancel{b^{11}}\,\cancel{c^{-16}}}{2^{2-1}\,\cancel{a^{-14}}\,b^{22-11}\,c^{-10+16}}$$

$$=\frac{a^{43}}{2^{1}\,b^{11}\,c^{6}}$$