PHASE & VERTICAL SHIFT / GRAPHS OF Sec x / Csc x

$$y = x^2 \qquad y = x^2 + 3 \qquad y = (x-5)^2$$
$$\text{vp } 3$$

$$y = 3\cos(4x + \pi) - 2$$
$$-2 + 3\cos(4x + \pi)$$

amp. 3

period $\dfrac{2\pi}{4} = \pi/2$

Vertical Shift -2

phase shift
(horiz. shift)

$$(4x + \pi) \qquad 4x + \pi = 0$$
$$4(x + \pi/4) \qquad 4x = -\pi$$
$$\text{left } \pi/4 \qquad x = -\dfrac{\pi}{4}$$
$$-\dfrac{\pi}{4}$$

$$\left(\dfrac{2}{3}x - \dfrac{5\pi}{6}\right)\dfrac{3}{2}$$
$$\dfrac{2}{3}\left(x - \dfrac{5\pi}{4}\right)$$

$$\dfrac{2}{3}x - \dfrac{5\pi}{6} = 0$$
$$\dfrac{3}{2} \cdot \dfrac{2}{3}x = \dfrac{5\pi}{6} \cdot \dfrac{3}{2}$$
$$x = \dfrac{5\pi}{4}$$

$$y = a \underline{\quad} (bx + c) + d$$

$\dfrac{\sin}{\cos}$

$\dfrac{\sec}{\csc}$ NA

amp $|a|$

period $\dfrac{2\pi}{b}$ $\dfrac{2\pi}{b}$

phase shift $bx + c = 0$ $bx + c = 0$ $bx + c = 0$

Vertical shift d d d

$y = -1 - 4\sin(2x - \pi)$

$\dfrac{\pi/2}{2\frac{\pi}{4}}$ $\dfrac{3\pi/4}{3\frac{\pi}{4}}$ $\dfrac{\pi}{4\frac{\pi}{4}}$ $\dfrac{5\pi/4}{5\frac{\pi}{4}}$ $\dfrac{3\pi/2}{6\frac{\pi}{4}}$

$\underline{amp}$ $\underline{period}$ $\underline{V.S.}$ $\underline{P.S.}$

$4(-)$ $\dfrac{2\pi}{2} = \pi$ -1 $2x - \pi = 0$

$\dfrac{2x}{2} = \dfrac{\pi}{2}$

$\underline{Spacing}$

$\pi \cdot \dfrac{1}{4} = \dfrac{\pi}{4}$

$y = \sin x$

$y = \csc x$
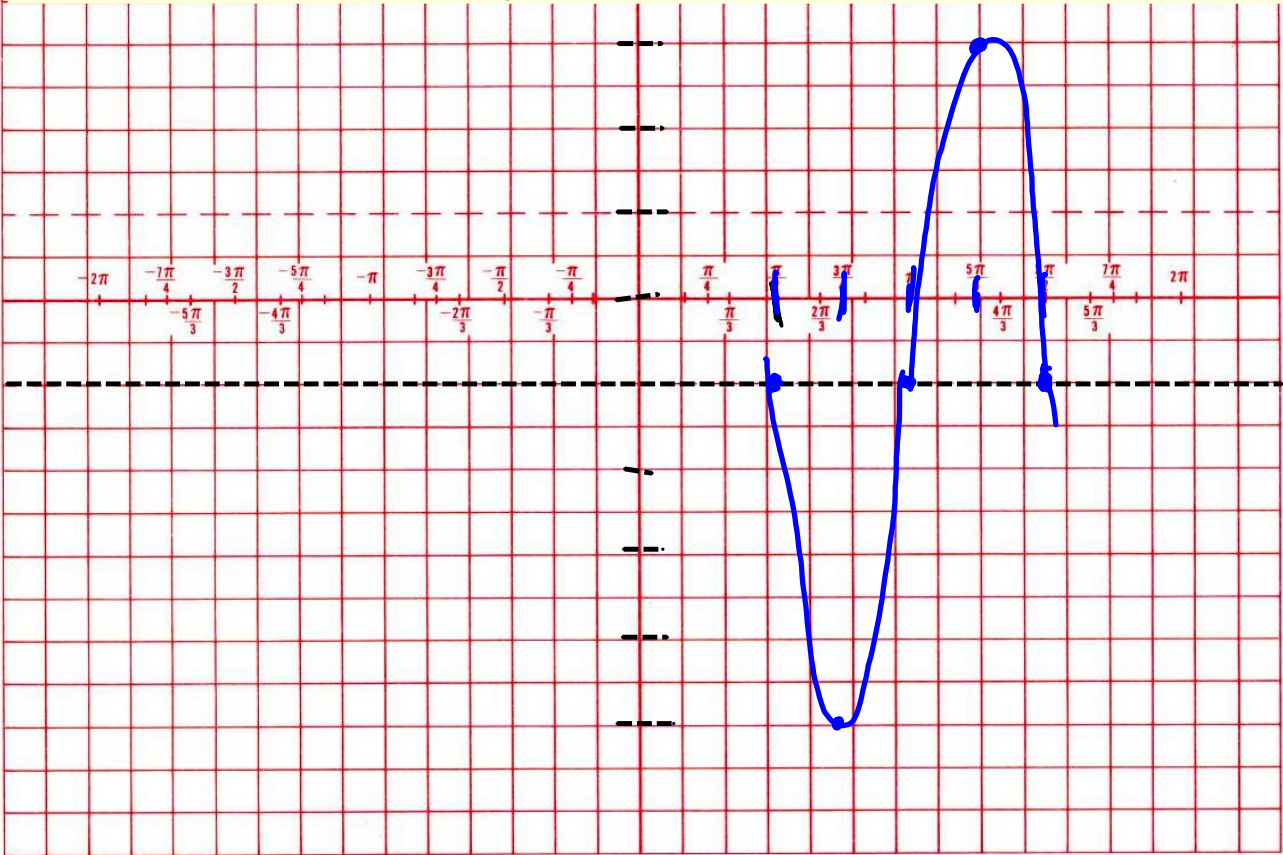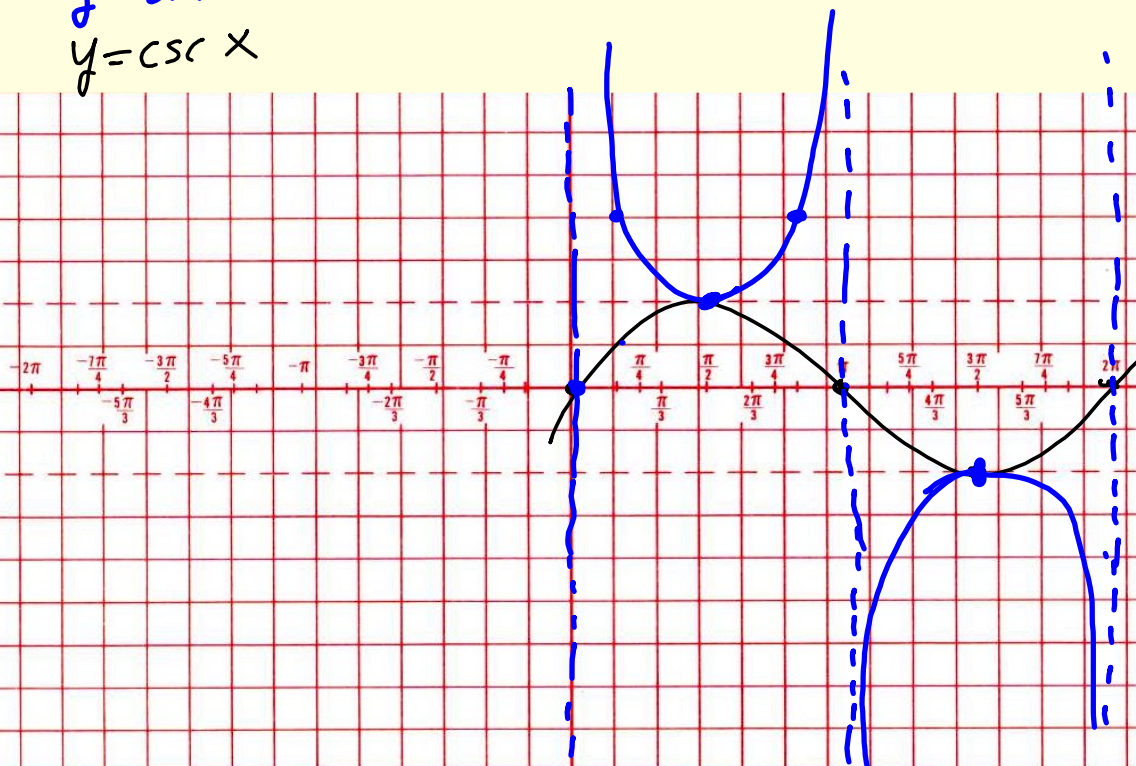
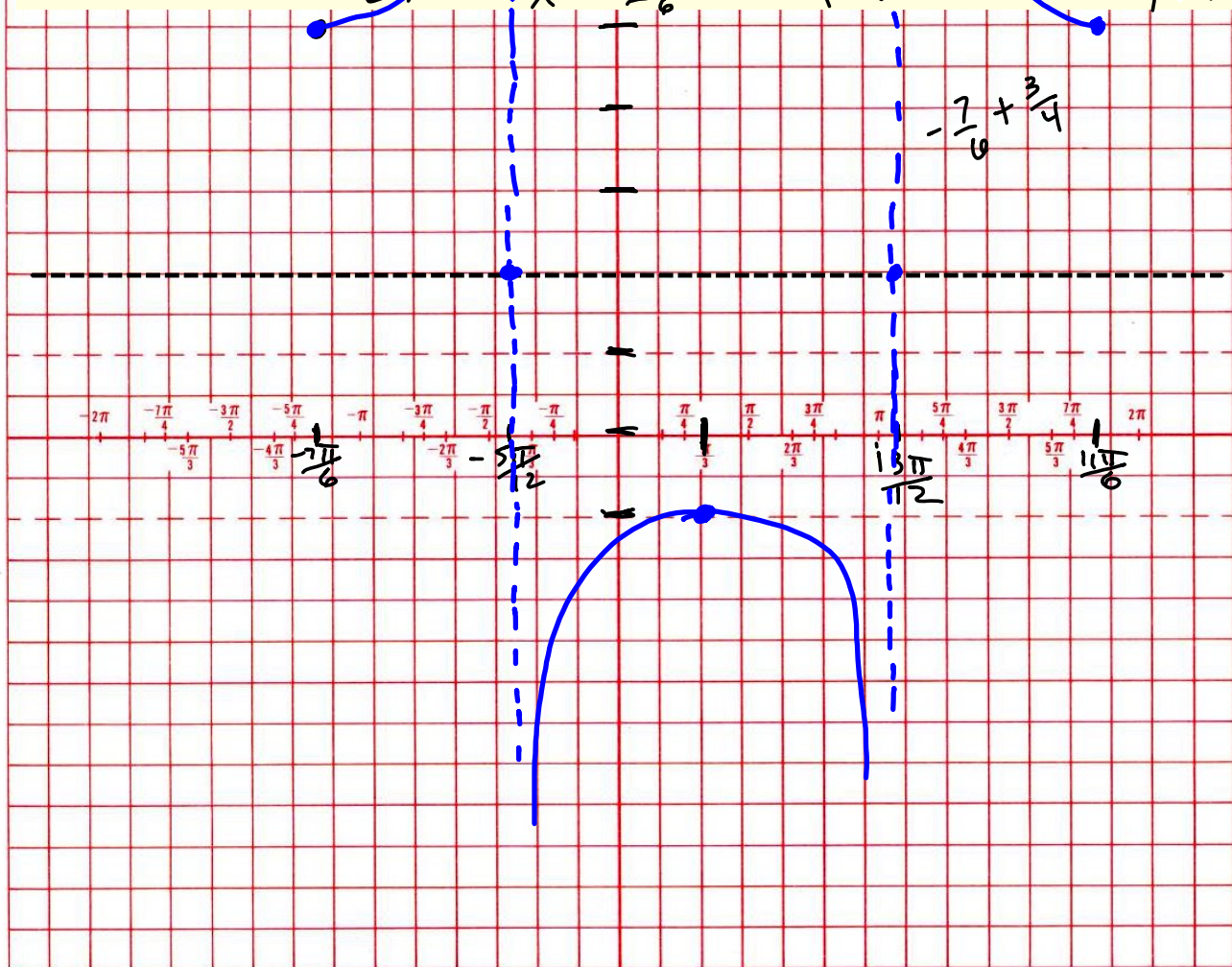$$y = 2 + 3\sec\left(\frac{2}{3}x + \frac{7\pi}{9}\right) \quad (\text{Like } \cos x)$$

$$-\frac{7\pi}{6} \quad -\frac{5\pi}{12} \quad \frac{\pi}{3} \quad \frac{13\pi}{12} \quad \frac{11\pi}{6}$$
$$-\frac{14\pi}{12} \quad -\frac{5\pi}{12} \quad \frac{4\pi}{12} \quad \frac{13\pi}{12} \quad \frac{22\pi}{12}$$

Amp
$$NA\,(3)$$

Period
$$\frac{2\pi}{\frac{2}{3}} = 2\pi \cdot \frac{3}{2}$$
$$= 3\pi$$

P.S.
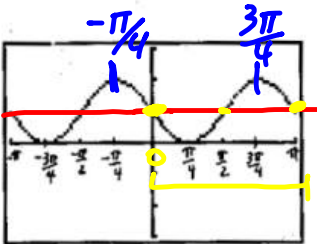$$\frac{2}{3}x = -\frac{7\pi}{9} \cdot \frac{3}{2}$$
$$x = -\frac{7\pi}{6}$$

V.S.
$$2$$

Spacing: $3\pi \cdot \frac{1}{4} = \frac{3\pi}{4} = \frac{9\pi}{12}$

$$-\frac{7}{6} + \frac{3}{4}$$

$$-2\pi \quad -\frac{7\pi}{4} \quad -\frac{3\pi}{2} \quad -\frac{5\pi}{4} \quad -\pi \quad -\frac{3\pi}{4} \quad -\frac{\pi}{2} \quad -\frac{\pi}{4} \quad \frac{\pi}{4} \quad \frac{\pi}{2} \quad \frac{3\pi}{4} \quad \pi \quad \frac{5\pi}{4} \quad \frac{3\pi}{2} \quad \frac{7\pi}{4} \quad 2\pi$$

$$-\frac{5\pi}{3} \quad -\frac{4\pi}{3} \quad -\frac{7\pi}{6} \quad -\frac{2\pi}{3} \quad -\frac{5\pi}{12} \quad \frac{\pi}{3} \quad \frac{2\pi}{3} \quad \frac{13\pi}{12} \quad \frac{4\pi}{3} \quad \frac{5\pi}{3} \quad \frac{11\pi}{6}$$

sin

cos

$-\pi/4$ $\dfrac{3\pi}{4}$

sec

tan

$\downarrow$ R-L

V.S = 1   period = π $\dfrac{3\pi}{4} + \dfrac{+\pi}{4} = \dfrac{9\pi}{4}$
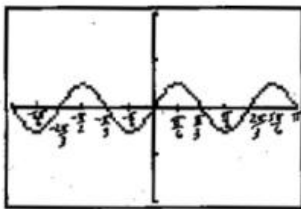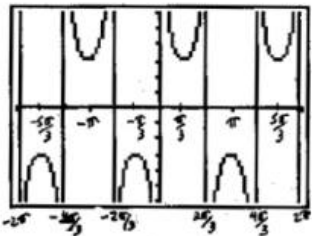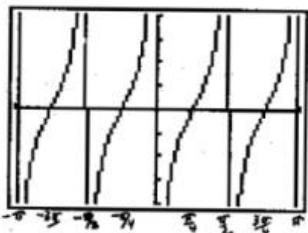
amp = 1   p.s. $-\dfrac{\pi}{4}$ $b = \dfrac{2\pi}{per} = \dfrac{2\pi}{\pi}$

$$y = 1\cos\left(2\left(x + \tfrac{\pi}{4}\right)\right) + 1$$
$$\left(2x + \pi/2\right)$$

csc X = sin x

V.S. O
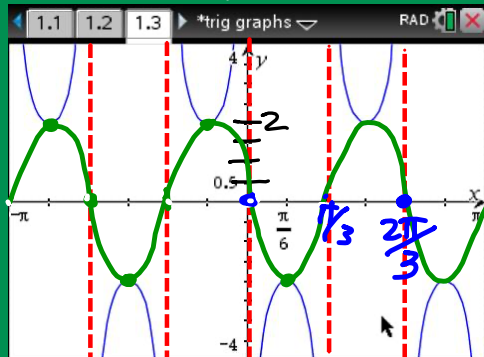
amp N/A (2)

$b = \dfrac{2\pi}{per} = \dfrac{2\pi}{2\pi/3}$ period $\dfrac{2\pi}{3}$

$= 2\pi \cdot \dfrac{3}{2\pi}$ p.s. O −

$\dfrac{\pi}{3}$ +

$y = 2\sec(3x)$

$2\sec\left(3\left(x - \pi/3\right)\right)$