RATIONAL EXPONENTS / SOLVING RADICAL EQUATIONS

Rule 6: $\sqrt[n]{a^m} = a^{m/n}$

$$\sqrt[5]{x^2} = x^{2/5}$$

The Hard Way

$$\sqrt[3]{x^1} \cdot \sqrt[5]{x}$$

$$x^{\frac{1}{3}} \cdot x^{\frac{1}{5}}$$

$$= x^{\frac{5}{15}} \cdot x^{\frac{3}{15}}$$

$$= x^{8/15} = \sqrt[15]{x^8}$$

$$\sqrt[4\cdot 3]{a^{3\cdot 3}b^{3}} \cdot \sqrt[3\cdot 4]{a^{4}b^{2\cdot 4}}$$

must make a common index

$$\sqrt[12]{a^9 b^3} \cdot \sqrt[12]{a^4 b^8}$$

$$= \sqrt[12]{a^{13} b^{11}}$$

$$a\sqrt[12]{a\, b^{11}}$$

EVALUATE. ← Answer is a #

$8^{1/3} = \sqrt[3]{8^1} = 2$

$81^{-1/2} = \frac{1}{81^{1/2}} = \frac{1}{\sqrt[2]{81^1}} = \frac{1}{9}$

$25^{3/2} = \sqrt[2]{25^3} = 5^3 = 125$

$\left(\frac{8}{125}\right)^{2/3} = \sqrt[3]{\left(\frac{8}{125}\right)^2} = \left(\frac{2}{5}\right)^2 = \frac{4}{25}$

$\left(\frac{49}{16}\right)^{-3/2} = \left(\frac{16}{49}\right)^{3/2} = \sqrt[2]{\left(\frac{16}{49}\right)^3} = \left(\frac{4}{7}\right)^3 = \boxed{\frac{64}{343}}$

Simplify.

$$\sqrt[3]{\sqrt[5]{x^7}} = \sqrt[15]{x^7}$$

$$\left(x^{7/5}\right)^{1/3} = x^{7/15}$$

$$\sqrt[m]{\sqrt[n]{a}} = \sqrt[m \cdot n]{a}$$

$$\sqrt[7]{\sqrt[3]{\sqrt[2]{x^4}}} = \sqrt[42]{x^4}$$

Quadratic Form

$ax^2 + bx + c$

$x^2 + 2x^1 - 3 = 0$

$(x+3)(x-1) = 0$

$x+3=0 \quad x-1=0$

$\boxed{x=-3 \quad x=1}$

$x^4 + 2x^2 - 3$

$x^6 - x^3 - 2$ yes

$x^9 - 2x^3 - 35$ no

$x^{2/3} + 3x^{1/3} - 10$ yes

$x^{4/5} - x^{2/5} - 2$ yes

$b^{2/5} - 1b^{1/5} - 6 = 0$

$(b^{1/5} + 2)(b^{1/5} - 3) = 0$

$b^{1/5} + 2 = 0 \qquad b^{1/5} - 3 = 0$

$b^{1/5} = -2 \qquad (b^{1/5})^5 = (3)^5$

$(\sqrt[5]{b})^5 = (-2)^5 \qquad b = 243$

$b = -32$

$(b^{1/3})^3 = (2)^3$

$(b^{\cancel{2/3}})^{3/2} = (4)^{3/2}$

$b = \sqrt[2]{4^3} = 2^3$

$= 8$

SOLVING RADICAL EQUATIONS

$\sqrt{x+4} + 2 = 5$

$\quad\quad\quad -2 \quad -2$

$\left(\sqrt{x+4}\right)^2 = (3)^2$

$x + 4 = 9$

$x = 5$

$\sqrt{5+4} + 2 = 5$

$\sqrt{9}$

$3 + 2 = 5$

$5 = 5$ ✓

Check: $\sqrt{16} - \sqrt{25} = -1$, $4 - 5 = -1$ ✓ | $\sqrt{0} - \sqrt{1} = -1$, $0 - 1 = -1$

1) Isolate a root.

$$\sqrt{2x-2} - \sqrt{3x-2} = -1$$

2) Square both sides.

$$\left(\sqrt{2x-2}\right)^2 = \left(\sqrt{3x-2} - 1\right)^2$$

FOIL!

$$2x-2 = \left(\sqrt{3x-2} - 1\right)\left(\sqrt{3x-2} - 1\right)$$

3) Clean up! Combine like terms.

$$2x-2 = 3x-2 - 1\sqrt{3x-2} - 1\sqrt{3x-2} + 1$$

4) Isolate the remaining root.

$$2x-2 = 3x - 1 - 2\sqrt{3x-2}$$

$$-2x+2 \qquad -2x+2$$

5) Square both sides.

$$\left(2\sqrt{3x-2}\right)^2 = (x+1)^2$$

FOIL

$$4(3x-2) = (x+1)(x+1)$$

6) Set = to 0 & solve (probably by factoring).

$$12x - 8 = x^2 + 2x + 1$$

$$-12x + 8$$

$$0 = x^2 - 10x + 9$$

$$0 = (x-9)(x-1)$$

$$x - 9 = 0 \qquad x - 1 = 0$$

$$\boxed{x = 9 \qquad x = 1}$$

7) Check answers in original problem. (See green at top of problem.)

Check answers!

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