Complex Fractions

$$\frac{\frac{1}{2}+\frac{2}{3}}{\frac{1}{5}+\frac{1}{2}} = \frac{\frac{3}{6}+\frac{4}{6}}{\frac{2}{10}+\frac{5}{10}} = \frac{\frac{7}{6}}{\frac{7}{10}} = \frac{\cancel{7}}{\cancel{6}_{3}} \cdot \frac{\cancel{10}^{5}}{\cancel{7}} = \frac{5}{3}$$

$$\frac{\frac{(x-2)\,x}{(x-2)(x+2)} - \frac{3\,(x+2)}{(x-2)(x+2)}}{\frac{(x+3)\,3x}{(x+3)(x-2)} - \frac{(x+2)(x-2)}{(x+3)(x-2)}} = \frac{\frac{x^2-2x-3x-6}{(x+2)(x-2)}}{\frac{3x^2+9x-(x^2-4)}{(x-2)(x+3)}} = \frac{\frac{x^2-5x-6}{(x+2)(x-2)}}{\frac{2x^2+9x+4}{(x-2)(x+3)}}$$

$$= \frac{x^2-5x-6}{(x+2)(x-2)} \cdot \frac{(x-2)(x+3)}{2x^2+9x+4}$$

$$= \frac{(x-6)(x+1)}{(x+2)\cancel{(x-2)}} \cdot \frac{\cancel{(x-2)}(x+3)}{(2x+1)(x+4)}$$

$$= \frac{(x-6)(x+1)(x+3)}{(x+2)(2x+1)(x+4)}$$

SOLVING RATIONAL EQUATIONS

Simplify

* results in an expression with variables.
* No = sign

Solve

* results in x=#
* has an = sign

$$\cancel{21}^{7}\left[\frac{x}{\cancel{3}} + {}^{\cancel{21}\,3}\frac{x}{\cancel{7}} = 21\cdot 2\right]$$

$$7x + 3x = 42$$

$$\frac{10x}{10} = \frac{42}{10}$$

$$x = \frac{21}{5}$$

$$\frac{x+5}{x^3+x^2} - \frac{2}{x^2-2x} = \frac{-3}{x^2-x-2}$$

1) Factor the denoms.

2) Check for excluded values.

$x \neq 0, -1, 2$

$$x^2(x+1)(x-2)\left[\frac{x+5}{x^2(x+1)} - \frac{2}{x(x-2)} = \frac{-3}{(x-2)(x+1)}\right]$$

$x^2(x+1)(x-2)$ $x^2(x+1)(x-2)$

3) Multiply by common denom & cancel all denoms.

$$(x+5)(x-2) - 2x(x+1) = -3x^2$$

$$\cancel{x^2} - 2x + 5x - 10 - \cancel{2x^2} - 2x = -3x^2$$

4) Write down terms that are left.

$$-x^2 + x - 10 = -3x^2$$
$$+3x^2$$

$$2x^2 + x - 10 = 0$$

5) Multiply & combine like terms

$$(2x+5)(x-2) = 0$$

6) Set = to 0 & solve.

$$x = \frac{-5}{2} \qquad \cancel{x = 2}$$

extraneous solution

7) Check excluded values.

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$$w + \frac{w+7}{w^2-3w-4} = \frac{w^2}{w-4} \qquad w \neq 4, -1$$

$$\cancel{(w-4)}\cancel{(w+1)} \qquad \cancel{(w-4)}(w+1)$$

$$(w-4)(w+1)\left[\frac{w}{1} + \frac{w+7}{\cancel{(w-4)}\cancel{(w+1)}} = \frac{w^2}{\cancel{w-4}}\right]$$

$$w(w-4)(w+1) + w + 7 = w^2(w+1)$$

$$w(w^2-3w-4)$$

$$w^3 - 3w^2 - 4w + w + 7 = w^3 + w^2$$

$$\cancel{w^3} - 3w^2 - 3w + 7 = \cancel{w^3} + w^2$$

$$+3w^2$$

$$0 = 4w^2 + 3w - 7$$

$$0 = (4w+7)(w-1)$$

$$\boxed{w = -\frac{7}{4} \qquad w = 1}$$