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Double & Half Angle Identities

$\cos 2A =$

$\cos(A+A) = \cos A \cos A - \sin A \sin A$

$\cos 2A = \cos^2 A - \sin^2 A$

$= 1 - 2\sin^2 A$

$= 2\cos^2 A - 1$

$1 - \sin^2 A - \sin^2 A$

$1 - 2\sin^2 A$

$\cos^2 A - \sin^2 A$

$\cos^2 A + (1 + \cos^2 A)$

$2\cos^2 A - 1$

$\sin 2A = \sin(A+A)$

$= \sin A \cos A + \cos A \sin A$

$= 2 \sin A \cos A$

$\tan 2A = \dfrac{\tan A + \tan A}{1 - \tan A \tan A} = \dfrac{2\tan A}{1 - \tan^2 A}$

Half Angle Identities

$\cos 2A = 2\cos^2 A - 1$

$\cos B = 2\cos^2 \frac{B}{2} - 1$

$\sqrt{\frac{1+\cos B}{2}} = \sqrt{\frac{2\cos^2 \frac{B}{2}}{2}}$

$\pm\sqrt{\frac{1+\cos B}{2}} = \cos \frac{B}{2}$

$\cos 2A = 1 - 2\sin^2 A$

$\cos B = 1 - 2\sin^2 \frac{B}{2}$

$\sqrt{\frac{2\sin^2 \frac{B}{2}}{2}} = \pm\sqrt{\frac{1-\cos B}{2}}$

$\tan \frac{A}{2} = \pm\sqrt{\frac{1-\cos A}{1+\cos A}\left(\frac{1+\cos A}{1+\cos A}\right)}$

$\sqrt{\frac{1-\cos^2 A}{(1+\cos A)^2}}$

$= \frac{\sin A}{1+\cos A}$

$\tan \frac{A}{2} = \frac{1-\cos A}{\sin A}$

T/F

T $\cos 50^\circ = 1 - 2\sin^2 25^\circ$ (A)

$= \cos 2(25^\circ)$

$= \cos 50^\circ$

F $\sin 42^\circ = 2\sin 89^\circ \cos 89^\circ$

$= \sin 2(89^\circ)$

$= \sin 168^\circ$

F $\cos 130^\circ = \sqrt{\frac{1-\cos 260^\circ}{2}}$

$= \sin\left(\frac{260^\circ}{2}\right)$

F $\cos 194^\circ = \sqrt{\frac{1+\cos 388^\circ}{2}}$

$= \cos\left(\frac{388^\circ}{2}\right)$

$= \cos 194^\circ$

Check Quad. where $\frac{A}{2}$ is located

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Evaluate. ← Answer is # probably special angle value

$$\frac{2\tan 75^\circ}{1-\tan^2 75^\circ} = \tan 2A$$

$$= \tan 2(75^\circ)$$

$$= \tan 150^\circ$$

$$= \boxed{-\frac{\sqrt{3}}{3}}$$

$$\frac{\sin 60^\circ}{1+\cos 60^\circ} = \tan\left(\frac{60^\circ}{2}\right)$$

$$= \tan 300^\circ$$

$$= \boxed{-\sqrt{3}}$$

Find $\sin 2x$ given $\tan x = \frac{2}{1} \frac{y}{x}$ & x in QIII

$\sin 2x = 2\sin x \cos x$

$= 2\left(\frac{-2}{\sqrt{5}}\right)\left(\frac{-1}{\sqrt{5}}\right)$ $\quad \frac{y}{r} \quad \frac{x}{r}$

$= \boxed{\frac{4}{5}}$

$1 + 4 = r^2$

$\sqrt{5} = \sqrt{r^2}$

Find $\cos \frac{A}{2}$ given $\sin A = -\frac{1}{2}$ A in QIII

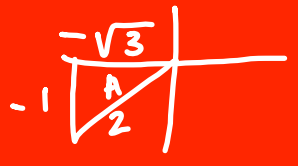
$\cos \frac{A}{2} = \pm\sqrt{\frac{1-\cos A}{2}}$ $\quad \frac{x}{r}$

$(-)\sqrt{\frac{\frac{2}{2} + \frac{\sqrt{3}}{2}}{2}}$

$-\sqrt{\frac{\frac{2+\sqrt{3}}{2}}{2}} \cdot \frac{1}{2}$

$-\sqrt{\frac{2+\sqrt{3}}{4}}$.

$\boxed{-\frac{\sqrt{2+\sqrt{3}}}{2}}$



$180° < A < 270°$

$90° < \frac{A}{2} < 135°$ ← II

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Verify $\dfrac{\sin 2x}{1-\cos 2x} = \cot x$

$\dfrac{2\sin x\cos x}{1-(1-2\sin^2 x)} = \dfrac{\cos x}{\sin x}$

$\dfrac{2\sin x\cos x}{2\sin^2 x}$

$\dfrac{\cos x}{\sin x} = \dfrac{\cos x}{\sin x}$

$\sin 4y = \cdots\cdots$

$\sin(2\cdot 2y) =$

$2A$

$2\sin 2A\cos 2A =$

$$1 - 8\sin^2\left(\frac{x}{2}\right)\cos^2\left(\frac{x}{2}\right) = \cos 2x$$

$$1 - 8\left(\sqrt{\frac{1-\cos x}{2}}\right)^2\left(\sqrt{\frac{1+\cos x}{2}}\right)^2 =$$

$$1 - 8\left(\frac{1-\cos x}{2}\right)\left(\frac{1+\cos x}{2}\right) =$$

$$1 - \frac{8}{4}\cdot\left(\frac{1-\cos^2 x}{4}\right)$$

$$1 - 2(\sin^2 x) = \cos 2x$$

$$1 - 2\sin^2 x = 1 - 2\sin^2 x$$

$\sin(4y) =$

$\sin(2\cdot 2y) =$

$2 \quad A$

$2\sin 2y \cos 2y$

$\sin 3x$

$\sin(x+2x)$