FUNDAMENTAL IDENTITIES

Identities - true for any value

$2(x+5) = 2x+10$

Trig Identities - true for any angle measure

Reciprocal

1. $\csc\theta = \frac{1}{\sin\theta}$

$\sin\theta = \frac{1}{\csc\theta}$

2. $\sec\theta = \frac{1}{\cos\theta}$

3. $\cot\theta = \frac{1}{\tan\theta}$

Ratio

4. $\tan\theta = \frac{\sin\theta}{\cos\theta}$

5. $\cot\theta = \frac{\cos\theta}{\sin\theta}$

Pythagorean

*6. $\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$

7. $1 + \tan^2\theta = \sec^2\theta$

8. $1 + \cot^2\theta = \csc^2\theta$

$\sin(-x) = -\sin x$

$\cos(-x) = \cos x$

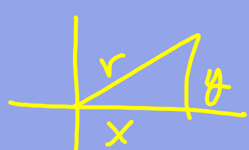
$\tan(-x) = -\tan x$

Even $f(-x) = f(x)$

Odd $f(-x) = -f(x)$

$y = \sin x$

$y = \cos x$



$\frac{x^2}{y^2} + \frac{y^2}{y^2} = \frac{r^2}{y^2}$

$\cot^2\theta + 1 = \csc^2\theta$

Simplify.

$(1+\tan x)^2 - 2\tan x$

$(1+\tan x)(1+\tan x) - 2\tan x$

$1 + \cancel{\tan x} + \cancel{\tan x} + \tan^2 x - \cancel{2\tan x}$

$= 1 + \tan^2 x$

$= \sec^2 x$

$\frac{(1+\cos x)}{(1+\cos x)}\frac{\cos x}{\sin x} + \frac{\sin x}{1+\cos x}\frac{(\sin x)}{(\sin x)}$

$= \frac{\cos x + \cancel{\cos^2 x + \sin^2 x}^{\,1}}{\sin x(1+\cos x)}$

$\frac{\cancel{\cos x + 1}}{\sin x\,\cancel{(1+\cos x)}}$

$= \boxed{\frac{1}{\sin x} \text{ OR } \csc x}$

Simplify

$$\frac{\sec^3 x - 8}{\sec^2 x - 4} = \frac{\cancel{(\sec x - 2)}(\sec^2 x + 2\sec x + 4)}{(\sec x + 2)\cancel{(\sec x - 2)}}$$

$$\frac{x^3 - 8}{x^2 - 4} = \frac{(x-2)(x^2+2x+4)}{(x+2)(x-2)}$$

$$= \frac{\sec^2 x + 2\sec x + 4}{\sec x + 2}$$

VERIFY.

WORK DOWNWARD!

$$\tan^2\theta \cdot \left(\frac{1}{\sec^2\theta}\right) \mp \cot\theta \tan(-\theta) = -\cos^2\theta$$

$$\tan^2\theta \cdot \cos^2\theta - \frac{1}{\cancel{\tan\theta}} \cdot \cancel{\tan\theta}$$

$$\frac{\sin^2\theta}{\cancel{\cos^2\theta}} \cdot \cancel{\cos^2\theta} - 1$$

$$\sin^2\theta - 1$$

$$-\cos^2\theta$$

$$= -\cos^2\theta$$

$$\frac{\sec\theta}{\sin\theta} - \frac{\sec\theta}{\csc(-\theta)} = \frac{1}{\tan\theta}$$

$$\frac{\frac{1}{\cos\theta}\cdot\frac{1}{\sin\theta}}{\cancel{\sin\theta}} - \frac{\frac{1}{\cos\theta}\cdot\frac{\sin\theta}{1}}{\frac{1}{\cancel{\sin\theta}}} = \frac{1}{\frac{\sin\theta}{\cos\theta}}$$

$$\frac{1}{\cos\theta\sin\theta} - \frac{\sin\theta\,\sin\theta}{\cos\theta\,\sin\theta} = \frac{\cos\theta}{\sin\theta}$$

$$\frac{1 - \sin^2\theta}{\cos\theta\,\sin\theta} =$$

$$\frac{\cos^{\cancel{2}}\theta}{\cancel{\cos\theta}\cdot\sin\theta}$$

$$\frac{\cos\theta}{\sin\theta} = \frac{\cos\theta}{\sin\theta}$$

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$$\cot^4 B - \csc^4 B = 1 - 2\csc^2 B$$

$$\cancel{(\cot^2 B - \csc^2 B)}(\cot^2 B + \csc^2 B) = 1 - 2\csc^2 B$$

$$(-1)(\cot^2 B + \csc^2 B) =$$

$$(-1)(\csc^2 B - 1 + \csc^2 B)$$

$$(-1)(2\csc^2 B - 1) =$$

$$-2\csc^2 B + 1 = 1 - 2\csc^2 B$$

$1 + \cot^2\theta = \csc^2\theta$

$\cot^2\theta - \csc^2\theta = -1$

$1 + \cot^2\theta = \csc^2\theta$

$\cot^2\theta = \csc^2\theta - 1$