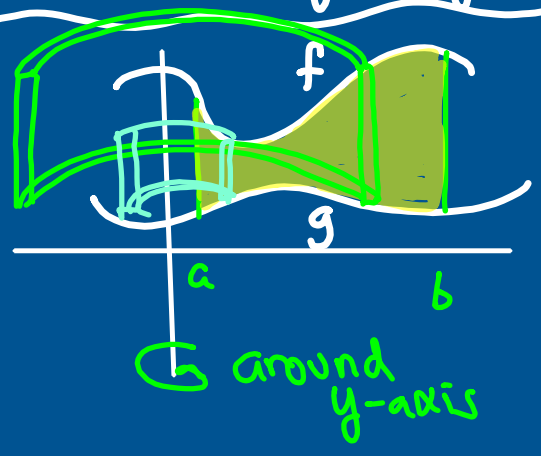
February 17, 2023

Volume By Cylindrical Shells



$V = l\,w\,dx$

$$2\pi \int_a^b r\,(f-g)\,dx$$

▯ is ∥ to the axis of revolution

▯ vertical

$y = x$'s

▯ horizontal

$x = y$'s

$y = x^3 \quad y = 1 \quad x = 2$

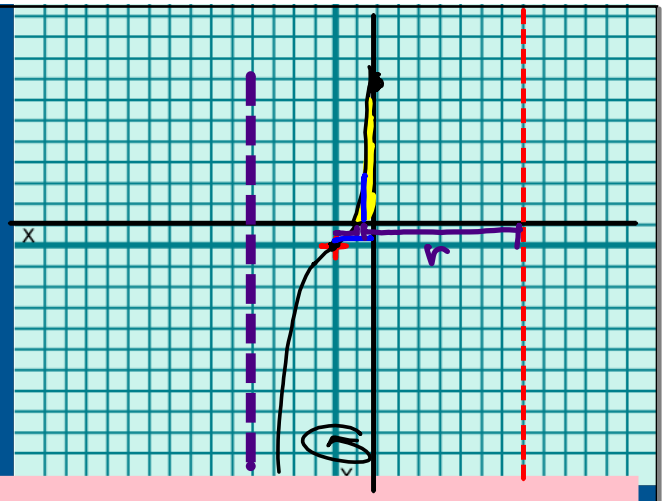
around y-axis

Top − Bottom ⇒ $y = x$'s

$$2\pi \int_a^b r\,(f-g)\,dx$$

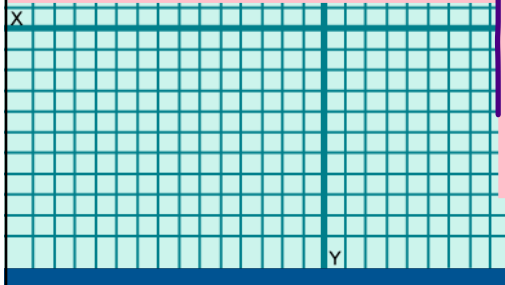
$$2\pi \int_1^2 x\,(x^3-1)\,dx$$

$$= \frac{47\pi}{5} \text{ units}^3$$



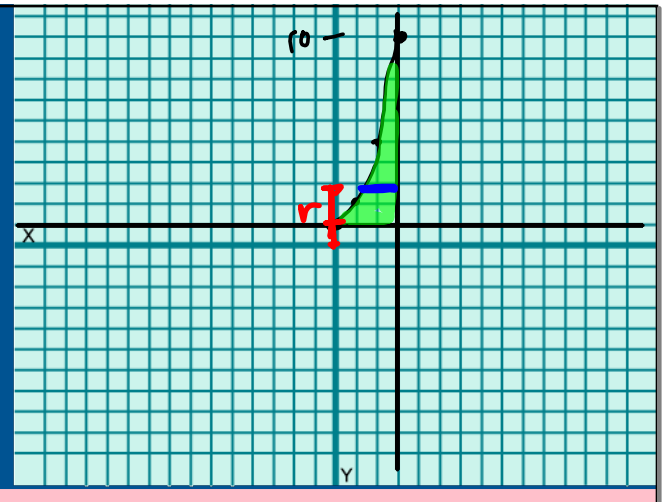
around $x=7$ $\quad 2\pi \int_1^2 (7-x)\,(x^3-1)\,dx$ (R−L, T−B)

around $x=-4$ $\quad 2\pi \int_1^2 (x+4)\,(x^3-1)\,dx$



$y = x^2 + 1 \quad y = 1 \quad x = 3$

around x-axis



Rect. horizontal $\Rightarrow x =$

$\sqrt{y-1} = \sqrt{x^2}$

$\pm \sqrt{y-1} = x \qquad x = 3$

$$2\pi \int_1^{10} y \left(3 - \sqrt{y-1}\right) dy$$

R − r

$$2\pi \int_1^{10} \left[3y - y\sqrt{y-1}\right] dy$$

$$2\pi \int_1^{10} 3y\, dy - 2\pi \int_1^{10} y\sqrt{y-1}\, dy$$

$u = y - 1 \quad u+1 = y$

$du = dy$

$$2\pi \cdot \frac{3y^2}{2}\Big|_1^{10} - 2\pi \int (u+1)u^{1/2}\, dy$$

$$-2\pi \int_0^9 \left(u^{3/2} + u^{1/2}\right) du$$

$$-2\pi \left[\frac{2}{5}u^{5/2} + \frac{2}{3}u^{3/2}\Big|_0^9\right.$$

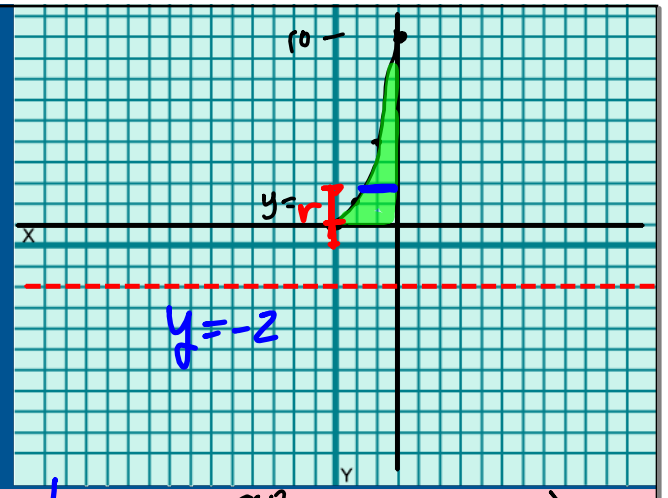
$y = x^2 + 1 \quad y = 1 \quad x = 3$

around x-axis

Rect. horizontal $\Rightarrow x =$

$\sqrt{y-1} = \sqrt{x^2}$

~~$\pm$~~ $\sqrt{y-1} = x \qquad x = 3$



$2\pi \int_1^{10} y \left(3 - \sqrt{y-1}\right) dy$

R - L

$2\pi \int_1^{10} (y+2)\left(3 - \sqrt{y-1}\right) dy$