Business Appl. of Diff. Eq.

The marginal revenue for digital watches is expressed by $\int \frac{dR}{dx} = \int 60{,}000 - \frac{40000x^{-2}}{x^2}$ dollars per thousand. If total sales revenue is $38,000 when 1000 watches are sold, what will revenue be for 4000 watches?

$$R = 60000x + \frac{40000x^{-1}}{+1} + C$$

$$R = 60{,}000x + \frac{40000}{x} + C$$

$$38000 = 60{,}000(1) + \frac{40000}{1} + C$$

$$-100{,}000$$

$$-62{,}000 = C$$

$$\boxed{R = 60{,}000x + \frac{40{,}000}{x} - 62{,}000}$$

$$R(4) = 240{,}000 + 10{,}000 - 62{,}000$$

$$= \$188{,}000$$

(b) $C(x) = 2000x^2 + \frac{40000}{x} + 20{,}000$

How many watches sold to maximize profit?
Max production is 20,000 watches.

$$R = 60{,}000x + \frac{40{,}000}{x} - 62{,}000$$

$P = R - C$

$P = 60{,}000x + \frac{40000}{x} - 62000 + \left[-2000x^2 - \frac{40000}{x} - 20{,}000\right]$

$* P = -2000x^2 + 60{,}000x - 82{,}000$

$P' = -4000x + 60000 = 0$

$\frac{60000}{4000} = \frac{4000x}{4000}$

$15 = x$

Optimization
1) Find crit. pts,
2) [,] or (,)
3) Test interval & crit pts.

$[0, 20]$

0	-82000
15	368,000
20	319,000

15,000 watches

$[0, \infty)$

$\lim_{x \to \infty} P(x)$

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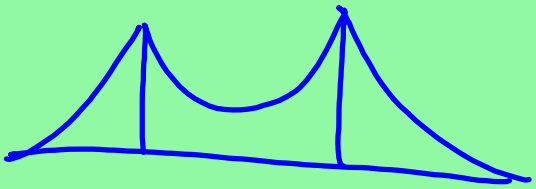
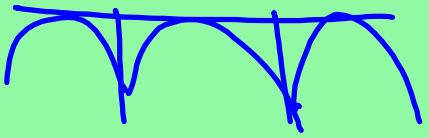
Hyperbolic Functions

- Combinations of e^x & e^{-x}
- properties of trig functions
- connected through complex numbers

$$\sinh x = \frac{e^x - e^{-x}}{2} \qquad \cosh x = \frac{e^x + e^{-x}}{2}$$



Catenary $y = a\cosh\left(\frac{x}{a}\right) + c$



$$\sinh(\ln 3) = \frac{e^{\ln 3} - e^{\ln 3^{-1}}}{2}$$

$$= \frac{3 - \frac{1}{3}}{2} = \frac{8}{3} \cdot \frac{1}{2} = \frac{4}{3}$$

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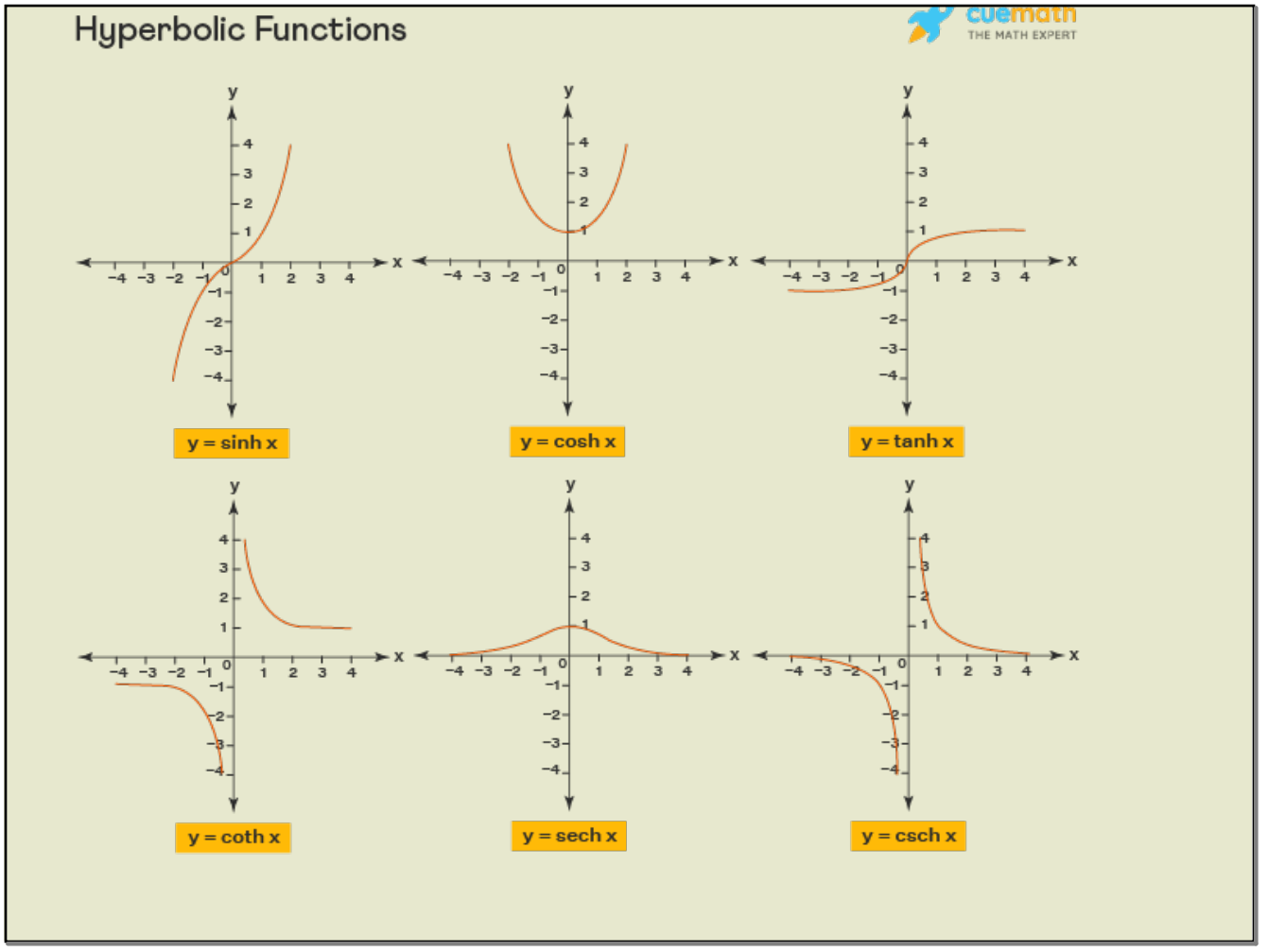
Hyperbolic Functions
cuemath
THE MATH EXPERT
y = sinh x
y = cosh x
y = tanh x
y = coth x
y = sech x
y = csch x

Derivatives

$\frac{d}{dx}\sinh x = \cosh x$ $\quad *\frac{d}{dx}\cosh x = \sinh x$ $\quad \frac{d}{dx}\cos x = -\sin x$

$\frac{d}{dx}\tanh x = \operatorname{sech}^2 x$ $\quad \frac{d}{dx}\coth x = -\operatorname{csch}^2 x$

$*\frac{d}{dx}\operatorname{sech} x = -\operatorname{sech} x \tanh x$ $\quad \frac{d}{dx}\operatorname{csch} x = -\operatorname{csch} x \coth x$

$\cosh^2 x - \sinh^2 x = 1$

$f(x) = \coth x \cdot e^{\operatorname{csch} x^3}$ $\quad$ Find $f'(x)$

$f'(x) = \coth x \cdot e^{\operatorname{csch} x^3} \cdot -\operatorname{csch} x^3 \coth x^3 \cdot 3x^2$

$+ e^{\operatorname{csch} x^3} \cdot -\operatorname{csch}^2 x$

$$\int \sinh^7 x \cosh x \, dx$$

$u = \sinh x$

$du = \cosh x \, dx$

$\frac{du}{\cosh x} = dx$

$$\int u^7 \cdot \cancel{\cosh x} \cdot \frac{du}{\cancel{\cosh x}}$$

$$= \frac{u^8}{8} + C$$

$$= \boxed{\frac{\sinh^8 x}{8} + C}$$