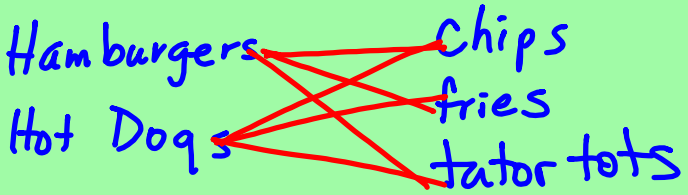
Combinatorics & Probability

of ways to perform an event



$2 \cdot 3 = 6$

How many different meals are possible?

Fundamental Counting Principle--If there are p ways to do one event and q ways to do another, then there are p · q ways to do both.

Car manufacturer

- 8 body colors
- 2 fabrics
- 3 option packages

How many different cars can be made?

$8 \cdot 2 \cdot 3 = 48$ cars

Permutations – # of arrangements or patterns that can be formed from a set of objects

Linear Permutations

1) All Objects = $n!$

How many ways are there to arrange the students sitting in the front row?

$4 \cdot 3 \cdot 2 \cdot 1 = 24$

factorial $4! = 24$

2) Arrange a small group chosen from a larger group $= {}_nP_r$

13 students $\quad 13 \cdot 12 \cdot 11 \cdot 10$

4 desks in front

${}_nP_r = {}_{13}P_4 = 17{,}160$

n = total # of objects, r = # to use

$${}_nP_r = \frac{n!}{(n-r)!}$$

$${}_{13}P_4 = \frac{13!}{9!} = \frac{13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdots 1}{9 \cdot 8 \cdot 7 \cdot 6 \cdots 1}$$

$${}_7P_2 = \frac{7!}{5!} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 42$$

How many ways can 4 relay runners be positioned for a race if chosen from 6 possible team members?

$${}_6P_4 = \frac{6!}{2!} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1} = 360$$

3) Alike Objects--indistinguishable, identical $= \frac{\text{total!}}{\text{alike! alike!}}$

How many permutations of the letters in the word MISSISSIPPI are possible?

$$\frac{11!}{4!\,4!\,2!} = 34{,}650$$

S I P

How many ways can 5 identical basketballs, 4 identical volleyballs, and 3 identical soccer balls be arranged in a line?

$$\frac{12!}{5!\,4!\,3!} = 27{,}720$$

Special Cases

4) Specific locations or Repeated objects – Draw blanks

How many ways can six people from this class be arranged in row if there must be a sophomore on each end of the row and two juniors in the middle seats?

$$\underset{So}{6} \cdot 9 \cdot \underset{Jr}{7} \cdot \underset{Jr}{6} \cdot 8 \cdot \underset{So}{5} = 90{,}720$$

How many different license plates are possible with 3 letters followed by 3 digits letters cannot be repeated but digits can be repeated?

$$\underset{L}{26} \cdot \underset{L}{25} \cdot \underset{L}{24} \cdot \underset{D}{10} \cdot \underset{D}{10} \cdot \underset{D}{10} = 15{,}600{,}000$$

COMBINATIONS – the # of groups that can be formed from a set of objects

$_nC_r$

$$_nC_r = \frac{n!}{(n-r)!\,r!}$$

$$_7C_5 = \frac{7!}{2!\,5!} = \frac{7\cdot\overset{3}{\cancel{6}}\cdot\cancel{5}\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot\cancel{1}}{\cancel{2}\cdot\cancel{1}\cdot\cancel{5}\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot\cancel{1}} = 21$$

Mrs. Meyer wants to form an "I Love Math" Committee to promote mathematics throughout the school. How many different committees of 5 students can be selected from this class?

$$_{13}C_5 = 1287$$

Card Facts

52 cards
26 red, 26 black
4 suits – ♢, ♤, ♡, ♧
13 cards in a suit
4 of ea. card
12 face cards

Draw 5 cards. How many hands of 5 diamonds are possible?

$_{13}C_5 = 1287$

How many hands with a full house are possible?

3 of a Kind + 2 of a Kind

$_4C_3 \cdot {}_{13}C_1 \cdot {}_4C_2 \cdot {}_{12}C_1$

$= 3744$

How many hands with 3 black cards and 2 red cards are possible?

$_{26}C_3 \cdot {}_{26}C_2 = 845{,}000$