Applications of Integration

Differential Equations

Complete/general solution
Leave C in solution

particular solution
Find value(s) of C

$$\int \frac{d^2y}{dx^2} = \int 24x^2 + 18x + 4$$

$$\frac{dy}{dx} = \frac{24x^3}{3} + \frac{18x^2}{2} + 4x + C$$

$$\int \frac{dy}{dx} = \int 8x^3 + 9x^2 + 4x + C$$

$$y = 2x^4 + 3x^3 + 2x^2 + C_1 x + C_2$$

$$\int \frac{d^2y}{dx^2} = \int (6x+2)^3 \, dx$$

$u = 6x+2$
$du = 6\,dx$
$\frac{du}{6} = dx$

$$\frac{dy}{dx} = \int u^3 \cdot \frac{du}{6}$$

$$= \frac{1}{6}\int u^3 \, du$$

$$= \frac{1}{6}\cdot\frac{u^4}{4} + C$$

$$\frac{dy}{dx} = \frac{1}{24}u^4 + C$$

$$= \frac{1}{24}(6x+2)^4 + C$$

$$\int \frac{dy}{dx} = \int \frac{1}{24}(6x+2)^4 + C$$

$$y = \frac{1}{24}\int u^4 \frac{du}{6} + \int C_1$$

$$y = \frac{1}{144}\cdot\frac{u^5}{5} + C_1 x + C_2$$

$$y = \frac{1}{720}(6x+2)^5 + C_1 x + C_2$$

Find a particular solution.

$\int \frac{d^2y}{dx^2} = \int 3x^2$

$y = -1$ when $x = 0$

$y' = 9$ when $x = 2$

$\frac{dy}{dx} = x^3 + C$

$9 = 2^3 + C$

$1 = C$

$\int \frac{dy}{dx} = \int x^3 + 1$

$y = \frac{x^4}{4} + x + C$

$-1 = 0 + 0 + C$

$\boxed{y = \frac{x^4}{4} + x - 1}$

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Motion

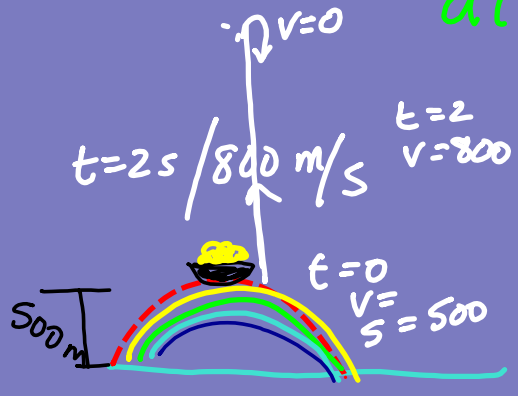
$s(t) =$

$v(t) = s'(t)$

$a(t) = v'(t) = s''(t)$

$a = -32\ ft/s^2$

$a = -9.8\ m/s^2$



$a(t) = -9.8$

$v(t) = -9.8t + C$

$800 = -9.8(2) + C$

$800 = -19.6 + C$

$819.6 = C$

$v = -9.8t + 819.6$

$v = -9.8t + 819.6$

$s(t) = -4.9t^2 + 819.6t + C$

$500 = 0 + 0 + C$

$s(t) = -4.9t^2 + 819.6t + 500$

Max height?

$0 = -9.8t + 819.6$

$9.8t = 819.6$

$t = 83.6$ sec

$s(83.6) = -$ ___ ___

When will it hit ground?

$s = 0$

$0 = -4.9t^2 + 819.6t + 500$

quadr. formula

$t =$ ___ sec

A bicyclist applies brakes + decelerate at 2 ft/s². How far will he travel before stopping if his speed has reduced to 6 ft/s after 2 sec?

$a = -2$

$a(t) = -2$

$v(t) = -2t + C$

$6 = -2(2) + C$

$6 = -4 + C$

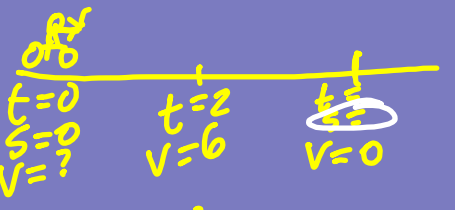
$10 = C$

$\boxed{v(t) = -2t + 10}$

$0 = -2t + 10$

$2t = 10$

$t = 5$ sec



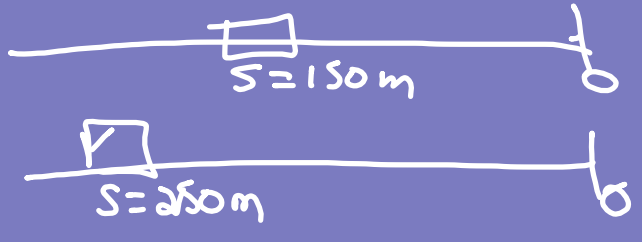
$s(t) = -t^2 + 10t + C$

$0 = 0 + 0 + C$

$\boxed{s(t) = -t^2 + 10t}$

$s(5) = -25 + 50$

$= \boxed{25 \text{ ft}}$



Business Applications

Marginal Revenue = change in rev per item sold

$$\frac{dR}{dx} = 60{,}000 - \frac{40{,}000x^{-2}}{x^2}$$ dollars per thousand.

Total sales revenue is $38,000 when 1000 watches are sold. What is revenue for 4000 watches?

(b) Cost of watches

$$C(x) = 2000x^2 + \frac{40000}{x} + 20,000$$

How many watches to optimize profit.

Max production level is 20,000 watches.