Polar Coordinates & Complex Numbers

Complex Numbers

$6+2i$

$0+4i$

$-2+0i$

$i^{83} = -i$

$\frac{83}{4} = 20.75$

$i = i$

$i^2 = -1$

$i^3 = -i$

$i^4 = 1$

$i = \sqrt{-1}$

$(2+5i) + (4-7i) = 6-2i$

$(3+5i)(2-4i)$

$= 6 - 12i + 10i + 20\cancel{i^2}$

$= \boxed{26-2i}$

$\frac{7-2i}{3+4i} \frac{(3-4i)}{(3-4i)}$

$= \frac{21-28i-6i+8\cancel{i^2}}{9+16\cancel{i^2}}$

$= \boxed{\frac{13-34i}{25}}$

$(3+2\sqrt{5}i)^8$

$(x^5)^{1/5} = (2+4i)^{1/5}$

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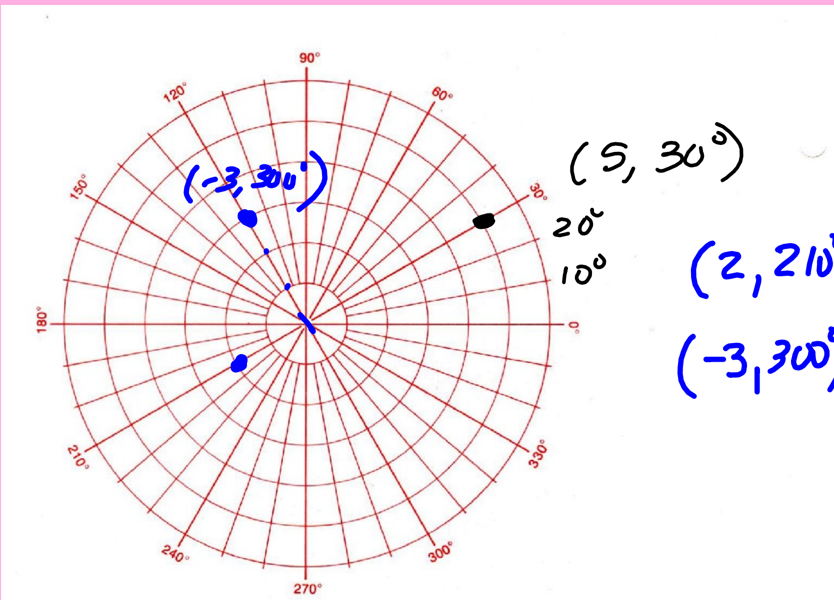
Polar Coordinates

(r, θ)



$(5, 30°)$

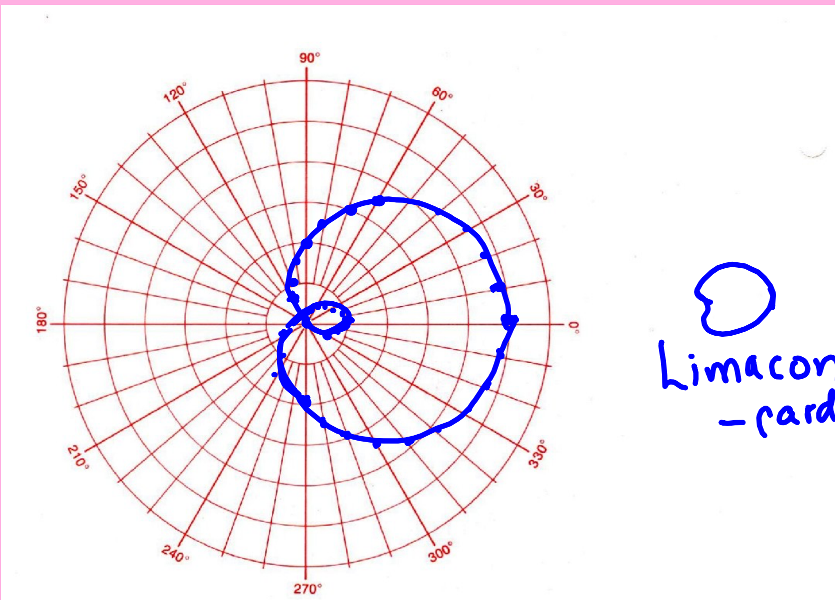
$20°$

$10°$

$(2, 210°)$

$(-3, 300°)$

$(-3, 300°)$



Limacon
- cardioid

$r = 2 + 3\cos\theta$

θ	r
0°	5
10°	4.95
20°	4.8
30°	4.6
	4.3
	4.2

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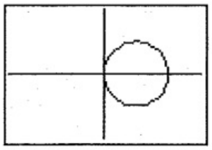
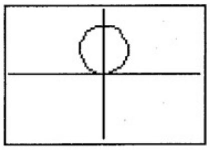
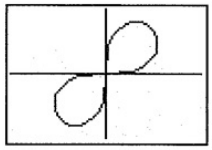
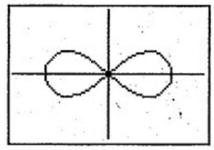
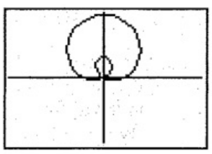
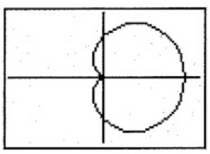
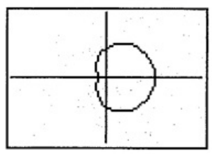
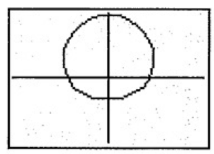
common polar graphs and forms of their equations. (In addition to circles, lemniscates, and roses just presented, we include *limaçons*. Cardioids are a special case of limaçons, where $|a/b| \geq 1$.)

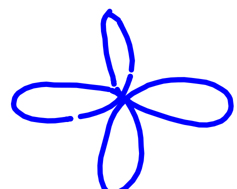
Circles and Lemniscates

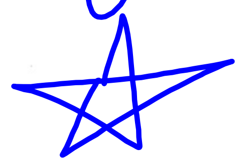
Circles		Lemniscates	
			
$r = a \cos \theta$	$r = a \sin \theta$	$r^2 = a^2 \sin 2\theta$	$r^2 = a^2 \cos 2\theta$

Limaçons

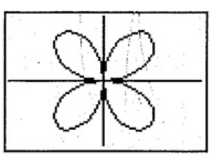
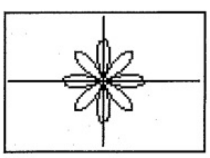
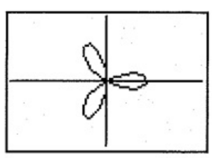
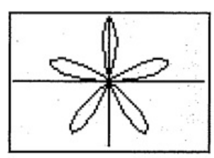
$r = a \pm b \sin \theta$ or $r = a \pm b \cos \theta$

			
$\frac{a}{b} < 1$	$\frac{a}{b} = 1$	$1 < \frac{a}{b} < 2$	$\frac{a}{b} \geq 2$





Rose Curves

$2n$ petals if n is even, $n \geq 2$		n petals if n is odd	
			
$n = 2$ $r = a \sin n\theta$	$n = 4$ $r = a \cos n\theta$	$n = 3$ $r = a \cos n\theta$	$n = 5$ $r = a \sin n\theta$

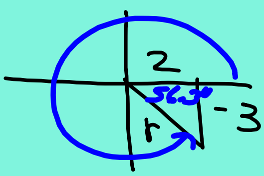
Converting between Equation Forms Sometimes an equation given in polar form is easier to graph in rectangular (Cartesian) form. To convert a polar equation to a rectangular equation, we use the following relationships, which were introduced in Section 8.2. See triangle POQ in Figure 36.

Converting Coordinates

Rectangular	Polar
(x, y)	(r, θ)

Rectangular → Polar

$(2, -3)$



$2^2 + 3^2 = r^2$

$4 + 9 = r^2$

$\sqrt{13} = r$

$r = \sqrt{x^2 + y^2}$

$\tan\theta = \frac{y}{x}$

$\tan\theta = -3/2$

$\tan^{-1}(3/2) = 56.3^\circ$

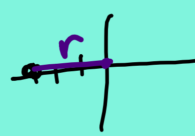
$\theta = 360^\circ - 56.3^\circ = 303.7^\circ$

$(r, \theta) = \boxed{(\sqrt{13}, 303.7^\circ)}$

$(-3, 0)$

$r = 3$

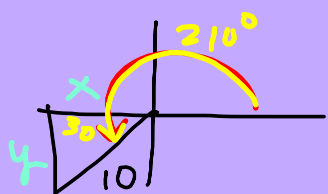
$\theta = 180^\circ$



$\boxed{(3, 180^\circ)}$

Polar $\rightarrow$ Rectangular

$(10, 210^\circ)$ Find (x, y)



$\cos\theta = \dfrac{x}{r}$ $\sin\theta = \dfrac{y}{r}$

$\boxed{r\cos\theta = x \qquad r\sin\theta = y}$

$x = 10\cos \overset{30}{210^\circ}$ $y = 10\sin 210^\circ$

$x = 10\left(-\dfrac{\sqrt{3}}{2}\right)$ $= 10\left(-\dfrac{1}{2}\right)$

$= -5\sqrt{3}$ $= -5$

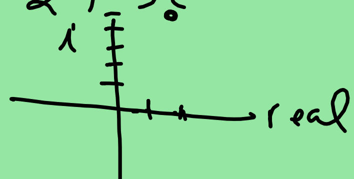
$\boxed{(-5\sqrt{3}, -5)}$

Complex #'s

Rectangular

$x + yi$

$2 + 5i$



Polar Form (Trigonometric Form)

$r\cos\theta + r\sin\theta\, i$

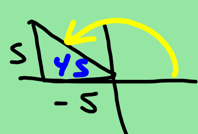
$= r(\cos\theta + i\sin\theta)$

$= r \text{ cis } \theta$

$2 \text{ cis } 45° = 2(\cos 45° + i\sin 45°)$

Rect → Polar

$-5 + 5i$



$25 + 25 = r^2$

$\sqrt{50} = \sqrt{r^2}$

$5\sqrt{2} = r$

$\tan\theta = \frac{5}{-5} = -1$

$\theta = 135°$

$\boxed{5\sqrt{2}(\cos 135° + i\sin 135°)}$

$3(\cos 30° + i\sin 30°)$

$3\left(\frac{\sqrt{3}}{2} + i\frac{1}{2}\right)$

$= \frac{3\sqrt{3}}{2} + \frac{3}{2}i$

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