Operations in Polar Form

$\times \quad \div \quad (\ )^2 \quad \sqrt[3]{\ }$

$2(\cos 30° + i\sin 30°) \cdot 5(\cos 70° + i\sin 70°)$

$10(\cos 30°\cos 70° + i\cos 30°\sin 70° + i\sin 30°\cos 70° + i^2\sin 30°\sin 70°)$

$10[\cos(30°+70°) + i\sin(30°+70°)]$

$\to 10(\cos 100° + i\sin 100°)$

$r_1(\cos\theta_1 + i\sin\theta_1) \cdot r_2(\cos\theta_2 + i\sin\theta_2) =$

$r_1 \cdot r_2(\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2))$

$37(\cos 211° + i\sin 211°) \cdot 4(\cos 348° + i\sin 348°)$

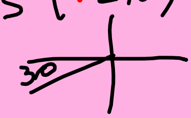
$211 + 348$

$148(\cos 559° + i\sin 559°)$

$$\frac{r_1(\cos\theta_1 + i\sin\theta_1)}{r_2(\cos\theta_2 + i\sin\theta_2)} = \frac{r_1}{r_2}\left(\cos(\theta_1-\theta_2) + i\sin(\theta_1-\theta_2)\right)$$

Divide & change to rectangular form. $340° - 550°$

$$\frac{15(\cos 340° + i\sin 340°)}{3(\cos 550° + i\sin 550°)} = 5\left(\cos(+210°) - i\sin(+210°)\right)$$



$$= 5\left(-\frac{\sqrt{3}}{2}\right) + i\left(+\frac{1}{2}\right)$$

$$= -\frac{5\sqrt{3}}{2} + \frac{5}{2}i$$

DeMoivre's Theorem

Polar Form better for: Raising complex # to a power)

$(2+5i\sqrt{3})^8$

$$[r(\cos\theta + i\sin\theta)]^3 = r^3(\cos 3\theta + i\sin 3\theta)$$

$$[r(\cos\theta + i\sin\theta)]^n = r^n(\cos(n\theta) + i\sin(n\theta))$$

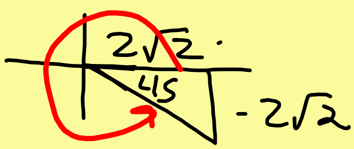
$$(2\sqrt{2} - 2i\sqrt{2})^6 = [4(\cos 315^\circ + i\sin 315^\circ)]^6$$

$$= 4^6(\cos(315^\circ\cdot 6) + i\sin(315^\circ\cdot 6)]$$

$$= 4096(\cos \underbrace{\cancel{1890}}_{90^\circ} + i\sin \underbrace{\cancel{1890}}_{90^\circ})$$

$$4096(0 + i\,1)$$

$$= \boxed{4096i}$$

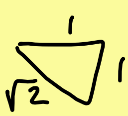


$(2\sqrt{2})^2 + (2\sqrt{2})^2 = r^2$

$8 + 8 = r^2$

$16 = r^2$

$4 = r$



$\tan\theta = \frac{2\sqrt{2}}{2\sqrt{2}}$

$= -1$

$\theta = 315^\circ$

Solve. $x^3 + 8 = 0$

#2) Finding all the roots of a complex #

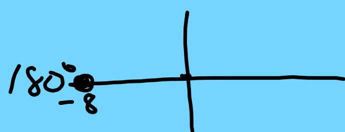
$(x^3)^{1/3} = (-8 + 0i)^{1/3} = \left[8(\cos 180^\circ + i\sin 180^\circ)\right]^{1/3} = 8^{1/3}(\cos 60^\circ + i\sin 60^\circ)$

$180 \cdot \frac{1}{3}$

$2\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$

$= \boxed{1 + i\sqrt{3}}$

$+360$



$r = 8$

$\theta = 180^\circ$

$\left[8(\cos 540^\circ + i\sin 540^\circ)\right]^{1/3} = 2(\cos 180^\circ + i\sin 180^\circ)$

$+360^\circ$

$= 2(-1 + i0)$

$= \boxed{-2}$

$\left[8(\cos 900^\circ + i\sin 900^\circ)\right]^{1/3} = 2(\cos 300^\circ + i\sin 300^\circ)$

60

$= 2\left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)$

$= \boxed{1 - i\sqrt{3}}$

March 22, 2023

1) Isolate the variable.

2) Eliminate the power on the variable by using the 1/n power.

3) Change to polar form.

4) Apply DeMoivre's Theorem.

5) Get additional answers by taking $(1/n) \cdot 360^\circ$ and add to first answer.

$x^4 - (-5-2i) = 0$

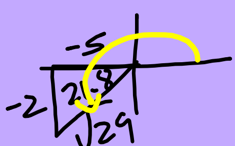
Find the 4th roots of $(-5-2i)$

$(x^4)^{1/4} = (-5-2i)^{1/4}$

$((29)^{1/2})^{1/4} = 29^{1/8}$

$\left[\sqrt{29}(\cos 201.8° + i\sin 201.8°)\right]^{1/4}$

$201.8 \cdot 1/4$



$25 + 4 = r^2$

$\sqrt{29} = \sqrt{r^2}$

$\tan\theta = \frac{-2}{-5}$

$\theta = 201.8$

$360 \cdot \frac{1}{4} = 90°$

$$\boxed{\begin{aligned}
&1)\ 29^{1/8}(\cos 50.45° + i\sin 50.45°) \quad +90\\
&2)\ 29^{1/8}\ \text{cis}\ 140.45°\\
&3)\ 29^{1/8}\ \text{cis}\ 230.45°\\
&4)\ 29^{1/8}\ \text{cis}\ 320.45°
\end{aligned}}$$