March 1, 2023

Solving Trig Equations

$2\sin^2 x + 5\sin x - 3 = 0$

$(2\sin x - 1)(\sin x + 3) = 0$

$2\sin x - 1 = 0$ $\quad$ $\sin x + 3 = 0$

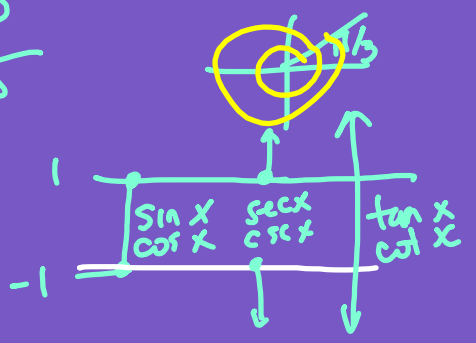
$\sin x = 1/2$ $\quad$ ~~$\sin x = -3$~~

$x = \pi/6, \frac{5\pi}{6}$

$[0, 2\pi)$

$2x^2 + 5x + 3 = 0$

$x = \frac{\pi}{3}, \frac{4\pi}{3}$



$\sec\theta = 2\cos\theta + 1 \qquad [0, 2\pi)$

$\cos\theta\left[\frac{1}{\cos\theta} = 2\cos\theta + 1\right]$

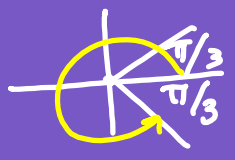
$1 = 2\cos^2\theta + \cos\theta$

$0 = 2\cos^2\theta + \cos\theta - 1$

$0 = (2\cos\theta - 1)(\cos\theta + 1)$

$2\cos\theta - 1 = 0 \qquad \cos\theta + 1 = 0$

$\cos\theta = \frac{1}{2} \qquad \cos\theta = -1$



$\theta = \frac{\pi}{3}, \frac{5\pi}{3}, \pi$

Check answers to make sure none cause division by 0.

~~$[0, 2\pi)$~~ $(-\infty, \infty)$

$\theta = \frac{\pi}{3} \pm 2\pi n$

$\theta = 5\pi/3 \pm 2\pi n$

$0 = \pi \pm 2\pi n$

$n = 1, 2, 3 \ldots$

$\theta = 30° \pm 360°n$

$(x \neq -3, 2)$

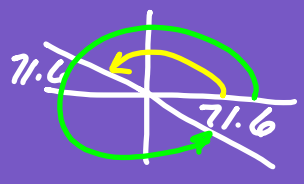
$\left[\frac{2}{x+3} + \frac{3}{x-2} = 1\right]$

$12\cot^2\theta - 5\cot\theta - 3 = 0 \quad [0°, 360°)$

$(3\cot\theta + 1)(4\cot\theta - 3) = 0$

$3\cot\theta + 1 = 0$

$\cot\theta = -\frac{1}{3}$



$\cot^{-1}(-1/3) = 108.4$

Don't use negative values!

$4\cot\theta - 3 = 0$

$\cot\theta = 3/4$

$\theta = 108.4°$
$= 288.4°$
$= 53.1°$
$= 233.1°$

$\sin\theta = -1/5$

$\sin^{-1}(-1/5) =$

$\sin^2\theta + \cos\theta = 0$ $\quad [0°, 360°)$

$1 - \cos^2\theta + \cos\theta = 0$ $\quad \sin^2\theta + \cos^2\theta = 1$

$0 = \cos^2\theta - \cos\theta - 1$

~~$0 = (\cos\theta + 1)(\cos\theta - 1)$~~

$\cos\theta = \dfrac{1 \pm \sqrt{1 + 4(1)(1)}}{2(1)}$ $\quad x^2 + x - 1 = 0$

$x = \dfrac{\quad}{\quad}$

$\cos\theta = \dfrac{1 \pm \sqrt{5}}{2}$

$\cos\theta = \dfrac{1+\sqrt{5}}{2}$

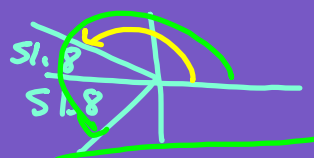
~~$\cos\theta = 1.618$~~

not possible

$\cos\theta = \dfrac{1-\sqrt{5}}{2}$

$\cos\theta = -0.618$

$\cos^{-1}(0.618\ldots)$



$\theta = 128.2, 231.8$