

MORE TRIG EQUATIONS

Square both sides -
Must check answers!

$$(2 \sin x)^2 = (1 - 2 \cos x)^2 \quad [0^\circ, 360^\circ]$$

$$(1 - 2 \cos x)(1 - 2 \cos x)$$

$$4 \sin^2 x = 1 - 4 \cos x + 4 \cos^2 x$$

$$4(1 - \cos^2 x) = 1 - 4 \cos x + 4 \cos^2 x$$

$$4 - 4 \cos^2 x = 1 - 4 \cos x + 4 \cos^2 x$$

$$0 = 8 \cos^2 x - 4 \cos x - 3$$

$$\cos x = \frac{4 \pm \sqrt{16 - 4(8)(-3)}}{2(8)} = \frac{4 \pm \sqrt{112}}{16}$$

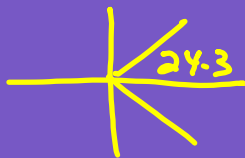
$$\cos x = 0.911$$

$$\cos^{-1}(0.911) = 24.3$$

$$\cos x = -0.411$$

$$\cos^{-1}(-0.411) = 115.3$$

Check!



$$\theta = \cancel{24.3^\circ}, 335.7^\circ, 115.3^\circ, \cancel{275.7^\circ}$$

$$[0, 2\pi) \quad \sin x - \cos 2x = 0$$

$$\sin x - (1 - 2\sin^2 x) = 0$$

$$\sin x - 1 + 2\sin^2 x = 0$$

$$2\sin^2 x + \sin x - 1 = 0$$

$$(2\sin x - 1)(\sin x + 1)$$

$$\sin x = \frac{1}{2} \quad \sin x = -1$$



$$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\cos 2x = 1 - 2\sin^2 x$$

$$\cos 2x = 2\cos^2 x - 1$$

$$\sin x = \cos\left(\frac{x}{2}\right) \quad [0^\circ, 360^\circ)$$

$$(\sin x)^2 = \left(\pm \sqrt{\frac{1 + \cos x}{2}}\right)^2 \quad \leftarrow \text{Check!}$$

$$\sin^2 x = \frac{1 + \cos x}{2}$$

$$1 - \cos^2 x = \frac{1 + \cos x}{2}$$

$$2 - 2\cos^2 x = 1 + \cos x$$

$$0 = 2\cos^2 x + \cos x - 1$$

$$0 = (2\cos x - 1)(\cos x + 1)$$

$$\cos x = \frac{1}{2} \quad \cos x = -1$$



$$x = 60^\circ, 300^\circ, 180^\circ$$

$$\sin 60^\circ = \cos 30^\circ$$

$$\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2} \quad \checkmark$$

$$\sin 300^\circ = \cos 150^\circ$$

$$-\frac{\sqrt{3}}{2} = -\frac{\sqrt{3}}{2} \quad \checkmark$$

$$\sin 180^\circ = \cos 90^\circ$$

$$0 = 0$$

$$[0, 2\pi) \quad 2 \tan\left(\frac{x}{2}\right) + \sqrt{3} = -\tan\left(\frac{x}{2}\right)$$

$$3 \tan\left(\frac{x}{2}\right) + \sqrt{3} = 0$$

$$\tan\left(\frac{x}{2}\right) = -\frac{\sqrt{3}}{3}$$



$$2. \quad \frac{x}{2} = -\frac{5\pi}{6}$$

$$x = \frac{5\pi}{3}$$

Shortcut
Do not use identities!

- 1) All same trig func
- 2) All same multiple angle

$$0 < x < 2\pi$$

$$0 < \frac{x}{2} < \pi$$

$$2 \cos^2 3\theta + 3 \cos 3\theta + 1 = 0 \quad [0^\circ, 360^\circ)$$

$$(2 \cos 3\theta + 1)(\cos 3\theta + 1) = 0$$

$$\cos 3\theta = -\frac{1}{2}$$



$$\cos 3\theta = -1 \quad \theta = [0^\circ, 360^\circ)$$

$$3\theta = [0^\circ, 1080^\circ)$$

$$\underline{3\theta} = \frac{120^\circ}{3}, \frac{180^\circ}{3}, \frac{240^\circ}{3}$$

$$480^\circ, 540^\circ, 600^\circ$$

$$840^\circ, 900^\circ, 960^\circ$$

$$\theta = 40^\circ, 60^\circ, 80^\circ$$

$$160^\circ, 180^\circ, 200^\circ$$

$$280^\circ, 300^\circ, 320^\circ$$

$$2 \sin^2 2\theta + \sin 2\theta - 1$$

$$2 \cos^2 2\theta + \sin 2\theta = 0$$