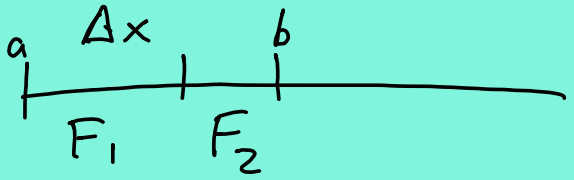
Work

Work = Force * distance
= 10 lb * 2 ft.
= 20 ft·lb

3 N·4 m = 12 N·m
= 12 J



$$W = \int_a^b F(x)\,dx$$

Well is 40 ft. deep
Bucket of water weighs 30 lb.
Loses $\frac{1}{4}$ lb for each ft.

1 hp = 550 $\frac{\text{ft·lb}}{\text{sec}}$

How much work is done?

$$\int_0^{40} \left(30 - \tfrac{1}{4}x\right) dx$$

$$30x - \tfrac{1}{8}x^2 \Big|_0^{40} = 1000 \text{ ft·lb.}$$

Crane raises bucket of sand weighing 500 lb. from ground to 50 ft in 10 sec. The cable weighs 5 lb/ft.

Sands falls out at 10 lb/sec

How much work to raise it from ground to 13.5 ft.?

$$\int_0^{13.5} \left[\underbrace{(500 - 2x)}_{\text{bucket}} + \underbrace{(250 - 5x)}_{\text{cable}} \right] dx$$

$$= 9487 \text{ ft} \cdot \text{lb}$$

$$\frac{10 \frac{\text{lb}}{\text{sec}}}{\frac{50 \text{ ft}}{10 \text{ sec}} = 5 \frac{\text{ft}}{\text{sec}}} = 2 \frac{\text{lb}}{\text{ft}}$$

$$\frac{10 \frac{\text{lb}}{\text{sec}} \cdot \frac{\text{sec}}{\text{ft}}}{5 \frac{\text{ft}}{\text{sec}}}$$

SPRINGS

Hooke's Law

$F(x) = kx$

- k: Spring constant
- x: # of units the spring is stretched or compressed from natural length.

Spring: natural length = 1 m

Force 8 N stretches it to 3 meters.

How much work is required to stretch it from 2 m to 4m?

1 to 3

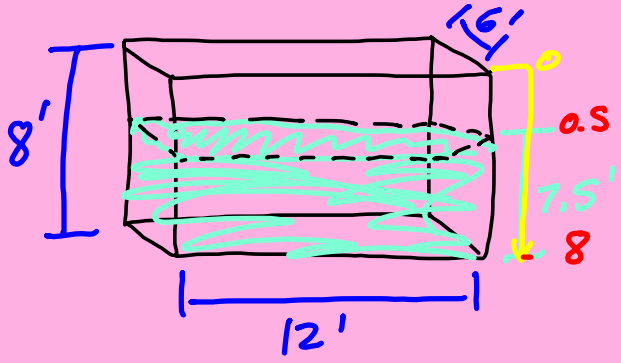
$F = kx$

$8 = k \cdot 2$

$4 = k$

$$\int_1^3 4x\,dx$$

$$= 2x^2 \Big|_1^3 = 18 - 2 = \boxed{16 \text{ J}}$$



How much work to pump all of the water out of the tank?

ρ = weight density = $62.4 \frac{lb}{ft^3}$

$= 9810 \frac{N}{m^3}$

$$\int \rho \cdot A_{l \cdot w} \cdot \text{depth}\ dx$$

bottom depth → 8, top of water → 0.5

$$\int_{0.5}^{8} 62.4 \cdot (12 \cdot 6) \cdot x \cdot dx = 143{,}208 \text{ ft} \cdot \text{lb}$$

Conical Tank

Water filled to 6 m.

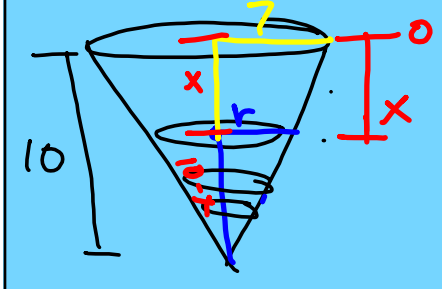
rho

$$\int \rho \cdot \underset{\text{Vol}}{A} \cdot \text{depth}\, dx$$

$$\int 9810 \cdot \pi r^2 \cdot x\, dx$$

$$9810\pi \int_4^{10} \left[\frac{7}{10}(10-x)\right]^2 x\, dx$$

$$\approx 5{,}980{,}123.4 \text{ J}$$



$$\frac{7}{10} = \frac{r}{10-x}$$

$$\frac{7}{10}(10-x) = r$$

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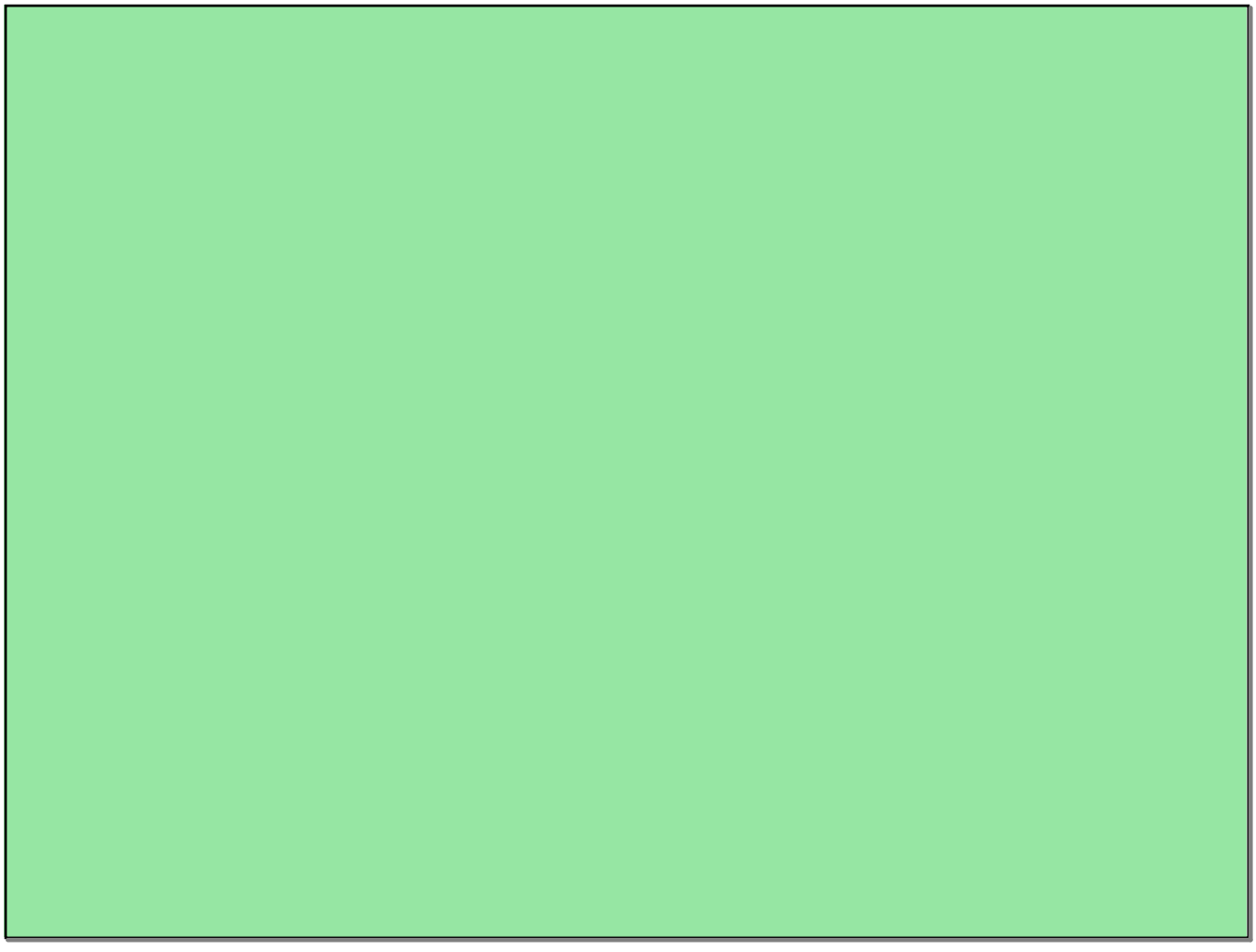

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