TECHNIQUES OF INTEGRATION

Integration by Parts

$$\int (f\cdot g)' = \int f\cdot g' + \int g\cdot f'$$

$$f\cdot g = \int f\cdot g' + \int g\cdot f'$$

$$f\cdot g - \int g\cdot f' = \int f\cdot g'$$

$$u\cdot v - \int v\cdot du = \int u\cdot dv$$

$$\boxed{\int u\cdot dv = u\cdot v - \int v\,du}$$

$$\int x^2 e^x\,dx$$

$$\int f\cdot g'$$

$$\int x \sec^2 x \, dx$$

$$\int u \, dv = u \cdot v - \int v \, du$$

$u = x$ $\quad$ $\int dv = \int \sec^2 x$

$du = dx$ $\quad$ $v = \tan x$

$$= x \tan x - \int \tan x \, dx$$

$$= x \tan x - \int \frac{\sin x}{\cos x} \, dx$$

$u = \cos x$

$du = -\sin x \, dx$

$\frac{du}{-\sin x} = dx$

$$= x \tan x + \int \frac{\cancel{\sin x}}{u} \cdot \frac{du}{+\cancel{\sin x}}$$

$$= x \tan x + \int \frac{1}{u} \, du$$

$$= x \tan x + \ln|u| + C$$

$$= \boxed{x \tan x + \ln|\cos x| + C}$$

$$\int \tan x \, dx = -\ln|\cos x| + C$$

$$\int \cot x \, dx$$

$$\int \frac{\cos x}{\sin x} \, dx$$

$u = \sin x$

$du = \cos x \, dx$

$$\int \frac{\cancel{\cos x}}{u} \cdot \frac{du}{\cancel{\cos x}}$$

$$= \ln|u| + C$$

$$\int \cot x \, dx = \ln|\sin x| + C$$

$$\int \ln x \, dx \qquad u = \ln x \quad \int dv = \int dx \quad uv - \int v\,du$$

$$du = \frac{1}{x}dx \quad v = x$$

$$= x\ln x - \int \cancel{x}\,\frac{1}{\cancel{x}}\,dx$$

$$= \boxed{x\ln x - x + C}$$

$$\int x^2 e^{2x}\,dx$$

$u = x^2$ $\quad dv = e^{2x}\,dx$

$du = 2x\,dx$ $\quad v = \frac{1}{2}e^{2x}$

$$\frac{1}{2}x^2e^{2x} - \int \cancel{2}x \cdot \frac{1}{\cancel{2}}e^{2x}\,dx$$

$$\frac{1}{2}x^2e^{2x} - \int x \cdot e^{2x}\,dx$$

$u = x$ $\quad dv = e^{2x}\,dx$

$du = dx$ $\quad v = \frac{1}{2}e^{2x}$

$$\frac{1}{2}x^2e^{2x} - \left[\frac{1}{2}xe^{2x} - \int \frac{1}{2}e^{2x}\,dx\right]$$

$$\frac{1}{2}x^2e^{2x} - \left[\frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x}\right] + C$$

$$\frac{1}{2}x^2e^{2x} - \frac{1}{2}xe^{2x} + \frac{1}{4}e^{2x} + C$$

$$\int e^{2x}\,dx$$

$u = 2x$

$du = 2\,dx$

$\frac{du}{2} = dx$

$$\int e^u \cdot \frac{du}{2}$$

$$\frac{1}{2}e^u + C$$

$$\frac{1}{2}e^{2x} + C$$

$$\int e^{5x}\,dx$$

$$\frac{1}{5}e^{5x} + C$$

$$\int \cos 3x\,dx$$

$$\frac{1}{3}\sin 3x + C$$

$$\int e^x \cos x \, dx$$

$u = e^x \quad dv = \cos x$

$du = e^x \, dx \quad v = \sin x$

$\left\{ \begin{array}{l} u = e^{3x} \\ du = e^{3x} \cdot 3 \end{array} \right.$

$$= e^x \sin x - \int e^x \sin x \, dx$$

$u = e^x \quad dv = \sin x$

$du = e^x \, dx \quad v = -\cos x$

$$\int e^x \cos x \, dx = e^x \sin x + +e^x \cos x \mp \int e^x \cos x \, dx$$

$$\frac{\cancel{2} \int e^x \cos x \, dx}{\cancel{2}} = \boxed{\frac{e^x \sin x + e^x \cos x}{2} + C}$$

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[redacted]