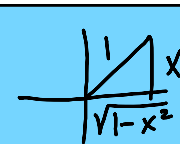
April 18, 2023

2) $$\int \frac{\sqrt{1-x^2}}{x^4}\,dx$$

$x = \sin\theta$

$dx = \cos\theta\, d\theta$



$$\int \frac{\sqrt{\cancelto{\cos^2\theta}{1-\sin^2\theta}}}{\sin^4\theta}\cos\theta\, d\theta$$

$$\int \frac{\cos^2\theta}{\sin^4\theta}\, d\theta$$

$$\int \frac{\cos^2\theta}{\sin^2\theta}\cdot\frac{1}{\sin^2\theta}\, d\theta$$

$$\int \cot^2\theta \csc^2\theta\, d\theta$$

$u = \cot\theta$

$du = -\csc^2\theta\, d\theta$

$$\int u^2\cdot\cancel{\csc^2\theta}\cdot\frac{du}{\cancel{-\csc^2\theta}}$$

$$= -\frac{u^3}{3} + C$$

$$-\frac{1}{3}\cot^3\theta + C$$

$$-\frac{1}{3}\left(\frac{\sqrt{1-x^2}}{x}\right)^3 + C$$

$$-\frac{(1-x^2)^{3/2}}{3x^3} + C$$

8/ $\int_0^2 \sqrt{16-4x^2}\,dx$

$x = 2\sin\theta$

$dx = 2\cos\theta\,d\theta$

$0 = 2\sin\theta$

$0 = \sin\theta$

$2 = 2\sin\theta$

$1 = \sin\theta$

$2\int_0^2 \sqrt{4-x^2}\,dx$

$2\int_0^{\pi/2} \sqrt{4-4\sin^2\theta} \cdot 2\cos\theta\,d\theta$

$8\int_0^{\pi/2} \sqrt{\underbrace{\cancel{1-\sin^2\theta}}_{\cos^2\theta}}\ \cos\theta\,d\theta$

$8\int_0^{\pi/2} \cos^2\theta\,d\theta$

$4\int_0^{\pi/2} (1+\cos 2\theta)\,d\theta$

$4\left[\theta + \frac{1}{2}\sin 2\theta\right|_0^{\pi/2}$

$\sin\pi$

$4\left[\frac{\pi}{2} + 0 - (0+0)\right]$

$= \boxed{2\pi}$

PARTIAL FRACTIONS

$$\int \frac{15x-14}{x^2-3x+2}\,dx \qquad = \frac{A}{x-1} + \frac{B}{x-2}$$

$(x-1)(x-2)$

$$(x-1)(x-2)\left[\frac{15x-14}{(x-1)(x-2)} = \frac{A}{\cancel{x-1}}(x-1)(x-2) + \frac{B}{\cancel{x-2}}(x-1)(x-2)\right]$$

$$15x-14 = A(x-2) + B(x-1)$$

$$15x-14 = Ax - 2A + Bx - B$$

$$15 = A + B$$

$$-14 = -2A - B$$

$$\begin{bmatrix} 1 & 1 \\ -2 & -1 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 15 \\ -14 \end{bmatrix} = \begin{array}{cc} \begin{bmatrix} -1 \\ 16 \end{bmatrix} & \begin{array}{c} A \\ B \end{array} \end{array}$$

$$\int \frac{-1}{x-1}\,dx \quad u = x-1, \ du = dx \qquad + \qquad \int \frac{16}{x-2}\,dx \quad u = x-2, \ du = dx$$

$$\int \frac{-1}{u}\,du \qquad\qquad \int \frac{16}{u}\,du$$

$$-\ln|u|$$

$$-\ln|x-1| \qquad + 16\ln|x-2| + C$$

$$\cancel{16}\ln|x-2|^{16} - \ln|x-1| + C$$

$$= \ln\left(\frac{|x-2|^{16}}{|x-1|}\right) + C$$

$$\int \frac{1}{4x-3}\,dx \qquad u = 4x-3, \quad du = 4\,dx, \quad \frac{du}{4} = dx$$

$$\int \frac{1}{u}\,\frac{du}{4}$$

$$\frac{1}{4}\ln|u|$$

$$\frac{1}{4}\ln|4x-3|$$

$$\int \frac{1-x^2}{4x^4+17x^2+4}\,dx$$

$$(4x^2+1)(x^2+4)$$

$$\frac{4x^3-3x^2+2}{x^2-3x-4}$$

1) Do Long division
2) Integrate quantity on top of the $\sqrt{\ }$
3) Do partial fractions on remainder.

$(4x^2+1)(x^2+4)$

$$\left[\frac{1-x^2}{(4x^2+1)(x^2+4)} = \frac{Ax^1+B}{4x^2+1} + \frac{Cx+D}{x^2+4}\right]$$

$$1-x^2 = (Ax+B)(x^2+4) + (Cx+D)(4x^2+1)$$

$$1-x^2 = Ax^3+4Ax+Bx^2+4B + 4Cx^3+Cx+4Dx^2+D$$

$$0 = A + 4C$$
$$-1 = B + 4D$$
$$0 = 4A + C$$
$$1 = 4B + D$$

$$\begin{bmatrix} 1 & 0 & 4 & 0 \\ 0 & 1 & 0 & 4 \\ 4 & 0 & 1 & 0 \\ 0 & 4 & 0 & 1 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1/3 \\ 0 \\ -1/3 \end{bmatrix}$$

$$\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}}$$
$$\frac{d}{dx}\tan^{-1}x = \frac{1}{x^2+1}$$
$$\frac{d}{dx}\sec^{-1}x = \frac{1}{|x|\sqrt{x^2-1}}$$

$$\int \frac{\frac{1}{3}}{4x^2+1}\,dx \quad \begin{matrix} u=2x \\ du=2\,dx \end{matrix} + \int \frac{-1/3}{x^2+4}\,dx$$

$$\frac{1}{3}\int \frac{1}{u^2+1}\cdot\frac{du}{2} \qquad + -\frac{1}{3\cdot 4}\int \frac{1}{\frac{x^2}{4}+1}\,dx \quad \begin{matrix} u=\frac{x}{2} \\ du=\frac{1}{2}dx \\ 2\,du=dx \end{matrix}$$

$$\frac{1}{6}\tan^{-1}u \qquad -\frac{1}{12}\int \frac{1}{u^2+1}\cdot 2\,du$$

$$\frac{1}{6}\tan^{-1}(2x)$$

$$-\frac{1}{6}\tan^{-1}u \quad + C$$

$$= \frac{1}{6}\tan^{-1}(2x) - \frac{1}{6}\tan^{-1}\left(\frac{x}{2}\right) + C$$

$$\frac{19x-32}{(4x+1)(2x-3)^2} = \frac{A}{4x^{1}+1} + \frac{B}{(2x-3)^2} + \frac{C}{2x-3}$$

$$\frac{}{x^2(x-3)} = \frac{A}{x^2} + \frac{B}{x} + \frac{C}{x-3}$$

$$(x^{1}-0)^2$$