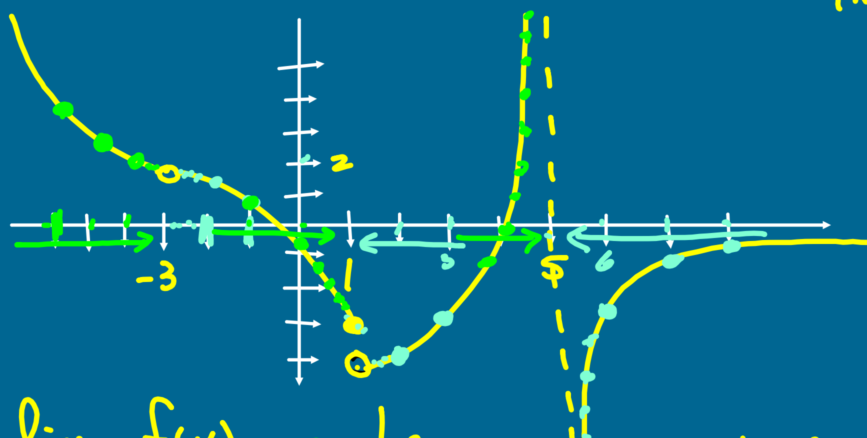
INTRO TO CALCULUS

- LIMITS
- DERIVATIVES
- INTEGRALS



Limits - find the y-coord as the x-coord approaches a given #.

$\lim_{x \to -3^-} f(x) = 2$

$\lim_{x \to -3^+} f(x) = 2$

$\lim_{x \to -3} f(x) = 2$

$f(-3) =$ undef.

$\lim_{x \to 1^-} f(x) = -3$

$\lim_{x \to 1^+} f(x) = -4$

$\lim_{x \to 1} f(x) =$ DNE

$f(1) = -3$

$\lim_{x \to 5^-} f(x) = +\infty$

$\lim_{x \to 5^+} f(x) = -\infty$

$\lim_{x \to 5} f(x) =$ DNE

$f(5) =$ undef.

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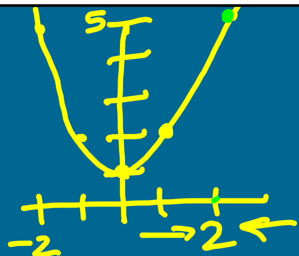
$$\lim_{x\to 2} \underline{x^2+1}$$
$$= 2^2+1$$
$$= \boxed{5}$$



Limits

1) Sub # in.

$$\lim_{x\to -4} \frac{x^2-3x}{\sqrt{x+8}} = \frac{(-4)^2-3(-4)}{\sqrt{-4+8}} = \frac{16+12}{\sqrt{4}} = \frac{28}{2} = 14$$

$$\lim_{x\to 3} \frac{x^2+4x-21}{x^2-3x} = \frac{9+12-21}{9-9} = \frac{0}{0}$$

indeterminate form

$$= \lim_{x\to 3} \frac{(x+7)\cancel{(x-3)}}{x\cancel{(x-3)}} = \frac{10}{3}$$

2) If $\frac{0}{0}$,

a) Factor

b) Conjugate

$$\lim_{x\to -5} \frac{x^3+125}{x^2-25} = \frac{-125+125}{25-25} = \frac{0}{0}$$

$$= \lim_{x\to -5} \frac{\cancel{(x+5)}(x^2-5x+25)}{\cancel{(x+5)}(x-5)} = \frac{25+25+25}{-5-5} = \frac{75}{-10} = \frac{15}{-2}$$

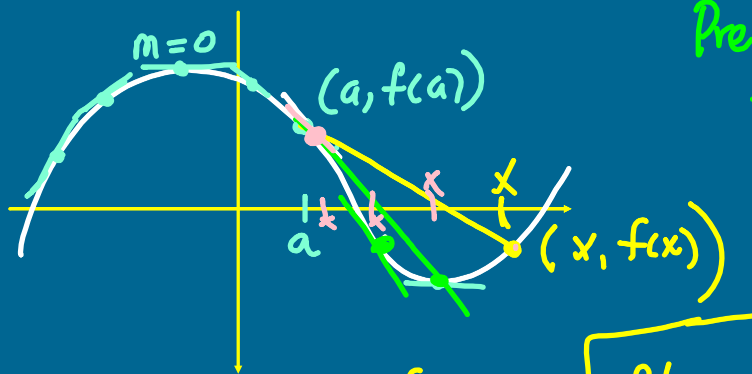
$$\frac{4}{3+\sqrt{2}}(3-\sqrt{2})$$

$$\lim_{x\to 64} \frac{\sqrt{x}-8}{x-64}\frac{(\sqrt{x}+8)}{(\sqrt{x}+8)} = \frac{8-8}{64-64} = \frac{0}{0}$$

$$\lim_{x\to 64} \frac{\cancel{x-64}}{\cancel{(x-64)}(\sqrt{x}+8)} = \frac{1}{\sqrt{64}+8} = \frac{1}{16}$$

DERIVATIVES

– represent the slope of a line tangent to a curve at a given point.



Pretend

$$f(x) = x^4 - 3x^3 + 2x^2 + x + 1$$

$$m = \frac{f(x) - f(a)}{x - a}$$

Def of Deriv.

$$\boxed{f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}}$$

$f(x) = 3x^2 + 4x - 5$ $\quad$ $f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

Find $f'(a)$.

$$\lim_{x \to a} \frac{\overbrace{3x^2 + 4x - \cancel{5}}^{f(x)} \overset{-}{+} \overbrace{(-3a^2 - 4a + \cancel{5})}^{f(a)}}{x - a} = \frac{0}{0}$$

$$\lim_{x \to a} \frac{(3x^2 - 3a^2) + (4x - 4a)}{x - a}$$

$$\lim_{x \to a} \frac{\overset{3(x^2 - a^2)}{3\cancel{(x-a)}(x+a)} + 4\cancel{(x-a)}}{\cancel{x-a}}$$

$$\lim_{x \to a} 3(x+a) + 4$$

Sub in a $\quad = 3(\overset{2a}{a+a}) + 4$

$= \boxed{6a + 4}$

b) $f(x) = \frac{1}{x^2}$

$$\lim_{x \to a} \frac{\frac{1}{x^2} - \frac{1}{a^2}}{x - a}$$

Make a common denom

$f(x)$	$f'(x)$
$3x^2+4x^1-5x^0$	$6x+4x^0$
* $5x^3-4x^7$	$15x^2-28x^6$
$\frac{1}{x^2}=x^{-2}$	$\frac{-2}{x^3}$ $\quad -2x^{-3}$

Power Rule

$$\frac{d}{dx}x^n = n\cdot x^{n-1}$$

↑ only used if:

1) all terms are + or −
2) all variables in numerator

Examples:

$$f(x) = 4x^8 + \frac{1}{2x^3} - 4x + 7 - \sqrt[3]{x}$$

$$= 4x^8 + \frac{1}{2}x^{-3-1} - 4x^1 + 7x^0 - x^{1/3-1}$$

$$f'(x) = 32x^7 - \frac{3}{2}x^{-4} - 4x^0 + 0 - \frac{1}{3}x^{-2/3}$$

$$= \boxed{32x^7 - \frac{3}{2x^4} - 4 - \frac{1}{3x^{2/3}}}$$

$$f(x) = (x^3-2x)(x^5+7x^2)$$ ← not + or − of terms

$$= x^8+7x^5-2x^6-14x^3$$ FOIL

$$\boxed{f'(x) = 8x^7+35x^4-12x^5-42x^2}$$