TRIG INTEGRALS

Use Sum & product identities

$$\int \sin 4x \cos 3x \, dx$$

$$\frac{1}{2}\int \left(\sin(4x+3x) + \sin(4x-3x)\right) dx$$

$$\frac{1}{2}\int \left(\sin 7x + \sin x\right) dx$$

$$\frac{1}{2}\left[-\frac{1}{7}\cos 7x - \cos x\right] + C$$

$$-\frac{1}{14}\cos 7x - \frac{1}{2}\cos x + C$$

$$\int_0^{\pi/2} \sin^5 x \, dx$$

$$\int \sin^4 x \cdot \sin x \, dx$$

$$\int (\sin^2 x)^2 \cdot \sin x \, dx$$

$$\int_0^{\pi/2} (1-\cos^2 x)^2 \cdot \sin x \, dx$$

$$\boxed{\begin{aligned} u &= \cos x \\ du &= -\sin x \, dx \\ \frac{du}{-\sin x} &= dx \end{aligned}}$$

$$\int_1^0 (1-u^2)^2 \cdot \cancel{\sin x} \cdot \frac{du}{-\cancel{\sin x}}$$

FOIL

$$-\int_1^0 (1 - 2u^2 + u^4) \, du$$

$u = \cos \pi/2 = 0$

$u = \cos 0 = 1$

$$-\left[u - \frac{2u^3}{3} + \frac{u^5}{5}\right]_1^0 + C$$

$$-\cos x + \frac{2}{3}\cos^3 x - \frac{1}{5}\cos^5 x + C$$

Odd Power

1) split of $\sin x$ or $\cos x$ from the odd powered term
2) Rewrite the remaining term as $(\sin^2 x)^p$ or $(\cos^2 x)^p$.
3) Use $\sin^2 x + \cos^2 x = 1$ to rewrite the squared term.
4) u-sub, simplify + integrate.

$$\int \sin^6 2x \cos^3 2x \, dx$$

$$\int \sin^6 2x \underline{\cos^2 2x} \cdot \cos 2x \, dx$$

$$\int \sin^6 2x \left(1 - \sin^2 2x\right) \cos 2x \, dx$$

$u = \sin 2x$

$du = 2\cos 2x \, dx$

$$\int u^6 (1 - u^2) \cancel{\cos 2x} \cdot \frac{du}{2\cancel{\cos 2x}}$$

$\frac{du}{2\cos 2x} = dx$

$$\frac{1}{2} \int (u^6 - u^8) \, du$$

$$\frac{1}{2}\left[\frac{u^7}{7} - \frac{u^9}{9}\right] + C$$

$$= \frac{1}{14} \sin^7 2x - \frac{1}{18} \sin^9 2x + C$$

$\cos 2x = 1 - 2\sin^2 x$

$\frac{2\sin^2 x}{2} = 1 - \cos 2x$

$\boxed{\sin^2 x = \frac{1}{2}(1 - \cos 2x)}$

$\int \sin^2 x \, dx =$

$\frac{1}{2}\int (1 - \cos 2x)\, dx$

$\frac{1}{2}\left[x - \frac{1}{2}\sin 2x\right] + C$

$\cos 2x = 2\cos^2 x - 1$

$1 + \cos 2x = \frac{2\cos^2 x}{2}$

$\boxed{\frac{1}{2}(1 + \cos \underset{4x}{2x}) = \cos^2 2x}$

$\int \cos^2 x \, dx$

$\frac{1}{2}\int (1 + \cos 2x)\, dx$

$\frac{1}{2}\left[x + \frac{1}{2}\sin 2x\right] + C$

$$\int \cos^4 x \, dx$$

$$\int (\cos^2 x)^2 \, dx$$

$$\int \left[\tfrac{1}{2}(1 + \cos 2x)\right]^2 dx$$

FOIL!

$$\frac{1}{4} \int (1 + 2\cos 2x + \cos^2 2x) \, dx$$

$$\frac{1}{4}\left[x + \cancel{2} \cdot \frac{1}{\cancel{2}} \sin 2x\right] + \frac{1}{4}\int (\cos^2 2x) \, dx$$

$$\frac{1}{4}x + \frac{1}{4}\sin 2x + \frac{1}{4}\int \frac{1}{2}(1 + \cos 4x) \, dx$$

$$\frac{1}{4}x + \frac{1}{4}\sin 2x + \frac{1}{8}\left[x + \frac{1}{4}\sin 4x\right] + C$$

$$\underline{\frac{1}{4}}x + \frac{1}{4}\sin 2x + \underline{\frac{1}{8}}x + \frac{1}{32}\sin 4x + C$$

$$\boxed{\frac{3}{8}x + \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x + C}$$

Both even powers

1) Rewrite as powers of $\sin^2 x$ or $\cos^2 x$
2) Use the double angle with $\sin^2 x$ or $\cos^2 x$
3) FOIL + integrate each term separately.

12

$$\int \sin^2 x \cos^4 x \, dx$$

$$\int \sin^2 x \left(\cos^2 x\right)^2 dx$$

$$\int \frac{1}{2}(1-\cos 2x) \cdot \left[\frac{1}{2}(1+\cos 2x)\right]^2 dx$$

$$\frac{1}{8}\int (1-\cos 2x)(1+\cos 2x)(1+\cos 2x)\, dx$$

$$\frac{1}{8}\int (1-\cos^2 2x)(1+\cos 2x)\, dx$$

$$\frac{1}{8}\int \sin^2 2x\,(1+\cos 2x)\, dx$$

$u =$

$$\frac{1}{8}\int \sin^2 2x \, dx + \frac{1}{8}\int \sin^2 2x \cos 2x \, dx$$