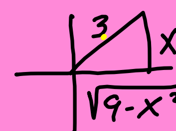
Trig Substitution

$a^2 - x^2$	$x = a\sin\theta$
$x^2 + a^2$	$x = a\tan\theta$
$x^2 - a^2$	$x = a\sec\theta$

$p^2 + x^2 = 9$

$\sqrt{p^2} = \sqrt{9-x^2}$



$\frac{y}{r}$ $\quad \frac{x}{3} = \sin\theta$

$\theta = \sin^{-1}\frac{x}{3}$

$x = 3\sin\theta$

$dx = 3\cos\theta\, d\theta$

$$\int \frac{x^2}{\sqrt{9-x^2}}\,dx$$

$$\int \frac{9\sin^2\theta}{\sqrt{9-9\sin^2\theta}} \cdot 3\cos\theta\, d\theta$$

$$\frac{27}{3}\int \frac{\sin^2\theta\cos\theta}{\sqrt{1-\sin^2\theta}}\,d\theta$$

$$9\int \frac{\sin^2\theta\cos\theta}{\sqrt{\cos^2\theta}}\,d\theta$$

$$9\int \frac{\sin^2\theta\cancel{\cos}\theta}{\cancel{\cos}\,\theta}\,d\theta$$

$$\frac{9}{2}\int (1-\cos 2\theta)\,d\theta$$

$$\frac{9}{2}\left[\theta - \frac{1}{2}\sin 2\theta\right] + C$$

Double angle identity

$$\frac{9}{2}\theta - \frac{9}{4}\sin 2\theta + C$$

$$\frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right) - \frac{9}{\cancel{4}}\cdot\cancel{2}\sin\theta\cos\theta + C$$

$$\frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right) - \frac{\cancel{9}}{2}\left(\frac{x}{\cancel{3}}\right)\left(\frac{\sqrt{9-x^2}}{\cancel{3}}\right) + C$$

$$\boxed{\frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right) - \frac{1}{2}x\sqrt{9-x^2} + C}$$

$1+\tan^2\theta=\sec^2\theta$

$$\int \tan^2\theta \, d\theta = \int (\sec^2\theta - 1)\, d\theta$$
$$= \tan\theta - \theta + C$$

$$\int \frac{1}{\sec^2\theta}\, d\theta = \int \cos^2\theta \, d\theta$$ (see $\cos^2\theta$ integration)

$$\int \frac{\cos\theta}{\sin^2\theta}\, d\theta =$$

$$\int$$

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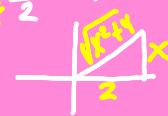
$$\int \frac{1}{x^2\sqrt{x^2+4}}\,dx$$

$x = 2\tan\theta$

$dx = 2\sec^2\theta\, d\theta$

$\rightarrow \frac{x}{2} = \tan\theta$



$$\int \frac{2\sec^2\theta}{4\tan^2\theta\sqrt{4\tan^2\theta+4}}\,d\theta$$

$$\frac{\cancel{2}}{4\cdot\cancel{2}}\int \frac{\sec^2\theta}{\tan^2\theta\sqrt{\tan^2\theta+1}}\,d\theta$$

$$\frac{1}{4}\int \frac{\sec^2\theta}{\tan^2\theta\sqrt{\sec^2\theta}}\,d\theta$$

$$\frac{1}{4}\int \frac{\sec^{\cancel{2}}\theta}{\tan^2\theta\cdot\cancel{\sec\theta}}\,d\theta$$

$$\frac{1}{4}\int \frac{\frac{1}{\cos\theta}}{\frac{\sin^2\theta}{\cos^2\theta}}\,d\theta$$

$$\frac{1}{4}\int \frac{1}{\cancel{\cos\theta}}\cdot\frac{\cos^{\cancel{2}}\theta}{\sin^2\theta}\,d\theta$$

$$\frac{1}{4}\int \frac{\cos\theta}{\sin^2\theta}\,d\theta$$

$u = \sin\theta$

$du = \cos\theta\, d\theta$

$\frac{du}{\cos\theta} = d\theta$

$$\frac{1}{4}\int \frac{\cancel{\cos\theta}}{u^2}\cdot\frac{du}{\cancel{\cos\theta}}$$

$$\frac{1}{4}\int \frac{1}{u^2}\,du$$

$$\frac{1}{4}\int u^{-2}\,du$$

$$\frac{1}{4}\,\frac{u^{-1}}{-1} + C$$

$$-\frac{1}{4u} + C$$

$$-\frac{1}{4u}\Big|$$

$$=\frac{-1}{4\sin\theta} + C$$

$$\frac{-1}{4\left(\frac{x}{\sqrt{x^2+4}}\right)} + C$$

$$= \boxed{-\frac{\sqrt{x^2+4}}{4x} + C}$$